

IG.0607

International Maths.

Functions - Graphs

Asymptotes

Notes

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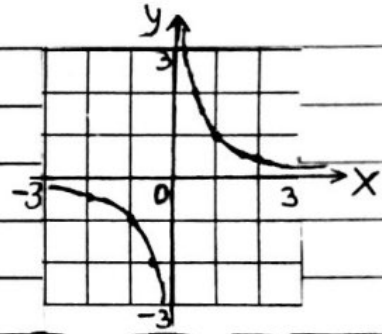
Functions (Graphs)AsymptotesGraph of Rational Algebraic functions,Case IWhen degree of Numerator $<$ degree of D° ,Example 1

$$f(x) = \frac{1}{x} \quad -3 \leq x \leq 3$$

(i) $x \rightarrow 0 \Rightarrow y \rightarrow \infty$

 \therefore y -axis ($x=0$) is an asymptote,

(ii) $x \rightarrow \infty \Rightarrow y \rightarrow 0$

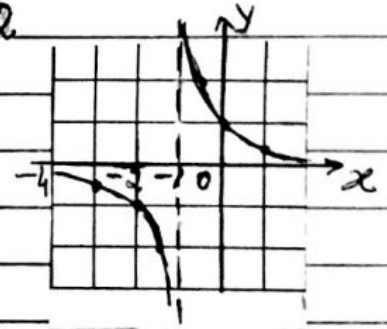
 x -axis ($y=0$) is an asymptote.Example 2

$$f(x) = \frac{1}{(x+1)} \quad -4 \leq x \leq 2$$

(i) $x \rightarrow -1 \Rightarrow y \rightarrow \infty$

 $x = -1$ is an asymptote.

(ii) $x \rightarrow \infty \Rightarrow y \rightarrow 0$

 x -axis ($y=0$) is another asymptote.Example 3

$$y = f(x) = \frac{x+2}{(x+1)(x-3)} \quad -5 \leq x \leq 5$$

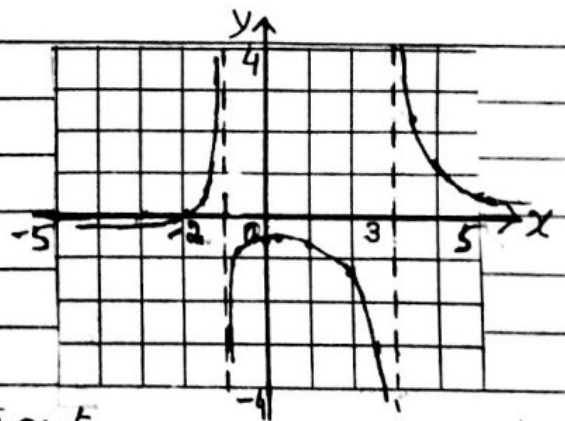
(i) $x \rightarrow -1 \Rightarrow y \rightarrow \infty$

 \therefore $x = -1$ is an asymptote.

(ii) $x \rightarrow 3 \Rightarrow y \rightarrow \infty$

 \therefore $x = 3$ (line \parallel y -axis) is asymptote.

(iii) $x \rightarrow \infty \Rightarrow y \rightarrow 0$

 \therefore x -axis ($y=0$) is an asymptote* (Note: Though the curve intersects the x -axis at $x = -2$)

Functions - Graphs/Asymptotes

classmate

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Case II

$$\underline{\text{Deg } N^{\circ} = \text{Deg } D^{\circ}}$$

Example 4.

$$f(x) = \frac{2x+1}{x}$$

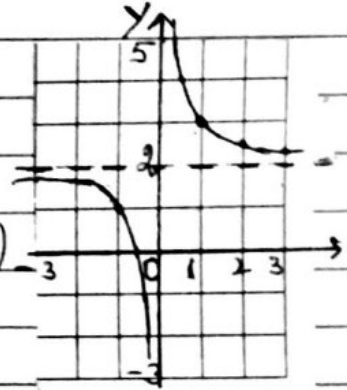
$$-3 \leq x \leq 3$$

$$\text{or } y = f(x) = 2 + \frac{1}{x}$$

Asymptotes are -

(i) y-axis (line $x=0$)

(ii) $y=2$ (a line parallel to x-axis)



Example 5.

$$y = f(x) = \frac{x^2 + 4x + 3}{x^2 - 4x + 3}$$

$$= 1 + \frac{8x}{(x-1)(x-3)}$$

$$-5 \leq x \leq 8$$

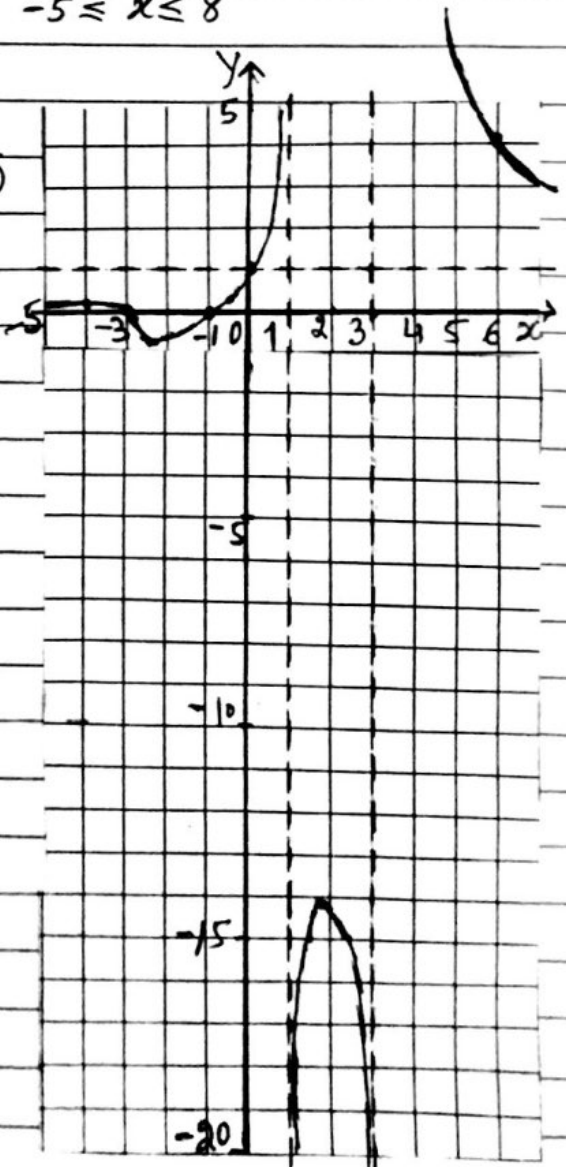
Asymptotes are

(i) $x=1$ and

(ii) $x=3$ (Vertical line)

(iii) $y=1$ (Horizontal line)

(Note: curve intersects the line $y=1$ at $(0,1)$)



Case III

Deg. $N^{\circ} > \text{Deg } D^{\circ}$

Example 6.

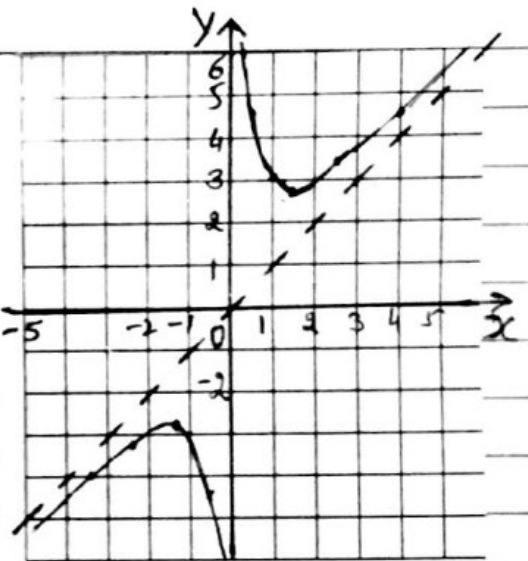
$$y = f(x) = \frac{x^2 + 2}{x}$$

$$\text{or } y = x + \frac{2}{x}$$

(i) $x \rightarrow 0 \Rightarrow y = \infty$

$x = 0$ (or y -axis) is asymptote.

(ii) $y = x$ is oblique asymptote.



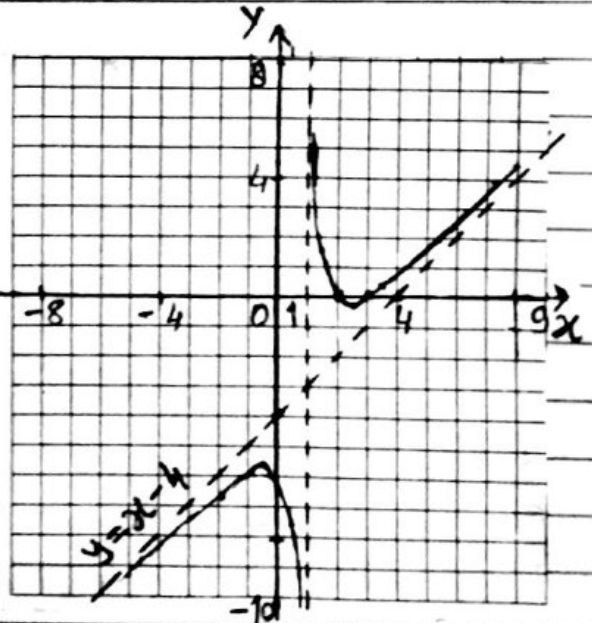
Example 7. $y = f(x) = \frac{x^2 - 5x + 6}{x - 1}$, $-8 \leq x \leq 8$

$$\text{or } y = (x - 4) + \frac{2}{(x - 1)}$$

(i) $x \rightarrow 1, y \rightarrow \infty$

$x = 1$ (Vertical) asymptote

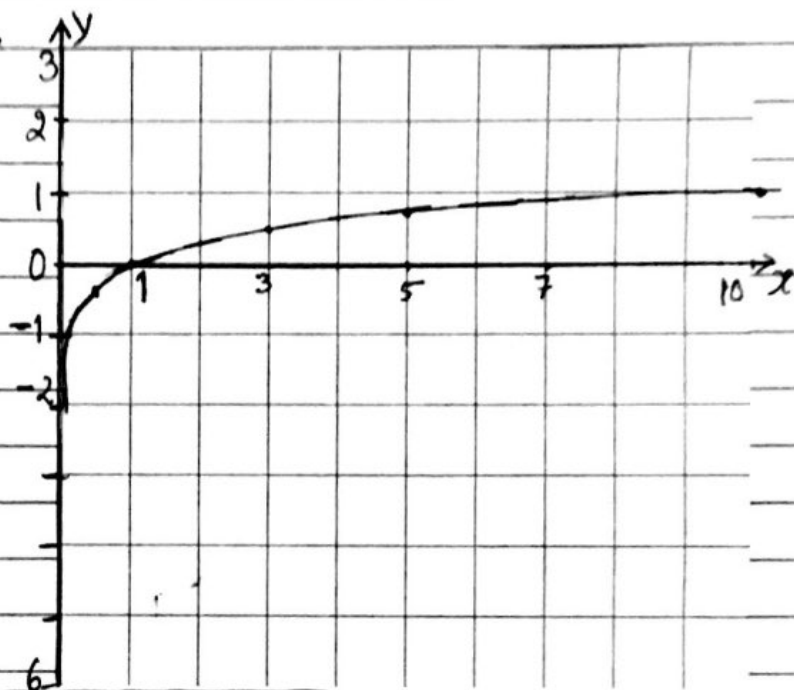
(ii) $y = (x - 4)$ another asymptote.



Logarithmic function.

Example 8. $y = \log x$

$x=0$ (y-axis)
is asymptote.



Example 9

Exponential function

$$y = f(x) = e^x$$

x-axis ($y=0$)
is the asymptote.

