Mechanics

Momentum

Exercise

Suresh Goel
Noida, Delhi, NCR,
India.
1. On a straight horizontal test track, driverless vehicles are tested. A car of mass 1600 kg is towing a trailer of mass 760 kg along the track. The brakes are applied, resulting in a deceleration of 12 m/s². The braking force acts on the car only. In addition to the braking force, there are constant resistance forces of 600 N on the car and 200 N on the trailer.
   (a) Find the magnitude of the force on the tow-bar. \[ 2 \]
   (b) Find the braking force. \[ 2 \]
   (c) At the instant when the brake are applied, the car has speed 25 m/s. At this instant the car is 17.5 m away from a stationary van, which is directly in front of the car. Show that the car hits the van at a speed of 8 m/s. \[ 2 \]
   (d) After the collision, the van starts to move with speed 5 m/s, and the car and trailer continue moving in the same direction with speed 20 m/s. Find the mass of the van. \[ M-20/42/Q6 \] \[ 3 \]

2. Three particles A, B, and C, of mass 2 kg, 3 kg, and 1 kg respectively, are in line with B and C at rest and A moving towards B with speed \( u \). The collision between A and B brings A to rest. B now collides with C. After this collision B and C move in the same direction, with C moving twice as fast as B. Find the final speeds of B and C in terms of \( u \).  

3. Particle A of mass 4 kg moves at 5 m/s towards particle B which is at rest. After a direct impact, A is at rest and B moves at 2 m/s. What is the mass of B. 

4. A arrow of mass 250 grams and travelling at 25 m/s hits directly and coalesces with a stationary target of mass 9.0 kg which can move freely. Calculate their velocity immediately after impact.
5. Particle A of mass 10 kg has speed 5 m/s. It collides directly with particle B of mass 2 kg moving in the opposite direction at 2 m/s. After the collision, A continues in the same direction with a speed of 3 m/s. Show that, if there are no further collisions, $m \leq 4$.

6. A particle P of mass 0.3 kg, lying on a smooth plane inclined at 30° to the horizontal, is released from rest. P slides down the plane for a distance of 2.5 m and then reaches a horizontal plane. There is no change in speed when P reaches the horizontal plane. A particle Q of mass 0.2 kg lies at rest on the horizontal plane 1.5 m from the end of the inclined plane. P collides directly with Q.

(a) It is given that the horizontal plane is smooth, and that, after the collision, P continues moving in the same direction, with speed 2 m/s.

Find the speed of Q after the collision. \[15\]

(b) It is given instead that the horizontal plane is rough and that when P and Q collide, the particles and move with speed 1.3 m/s.

Find the coefficient of friction between P and the horizontal plane. \[520/241/87\] \[15\]
7. Small smooth spheres A and B, of equal radii and of masses 4 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed 10ms⁻¹. After the spheres collide A continues to move in the same direction but with half the speed of B.

(a) Find the speed of B after the collision. -- [2]

A third small smooth sphere C, of mass 1 kg and with the same radius as A and B, is at rest on the plane. B now collides directly with C. After the collision B continues to move in the same direction but with one third the speed of C.

(b) Show that there is another collision between A and B. -- [3]

(c) A and B coalesce during this collision. Find the total loss of kinetic energy in the system due to the three collisions. -- [5]

8. Particles P of mass m kg and Q of mass 0.2 kg are free to move on a smooth horizontal plane. P is projected at a speed of 2ms⁻¹ towards Q which is stationary. After the collision P and Q move in opposite directions with speeds of 0.5 m s⁻¹ and 1m s⁻¹ respectively. Find m. -- [3]
M.1

Momentum

1. (a) \[ \text{Trailer: } -T - 200 = 700 \times 12 \] 
2. \[ \text{Car: } +T - 100 - F = 1600 \times 12 \] 

System: 
\[ -600 - 200 - F = 2360 \times 12 \]

\( T \) = 8200 N

\( \text{Magnitude: } \frac{V}{T} = 0.676 \text{ m/s} \)

(b) \[ \text{Car} \rightarrow \text{Trailer} \]

\[ \text{Trailer} \rightarrow \text{Car} \]

\( a = -12 \)

\[ \text{System: } -600 - 200 - F = 2360 \times 12 \]

\( \text{Breaking force: } F = 26800 \times 14 \sqrt{7} \)

\( V = 8 \text{ m/s} \)

(c) \[ V^2 = u^2 + 2as \]

\( V^2 = 22^2 + 2(-12) \times 17 \cdot 5 \)

\( \Rightarrow V = 8 \text{ m/s} \)

\( V_B \) for no further collision

\( V_B \geq 3 \)

\( 20 - 2v \geq 3 \)

\( 20 - 2m \geq 3 \)

\( m \leq 8 \)

6. \[ 0.3g \sin 30^\circ = 30 \Rightarrow a = 5 \]

(a) \[ V^2 = 0 + 2 \times 2 \times 15 \times a \]

\( = 5 \times 5 \Rightarrow V = 5 \text{ m/s} \)

Now \( u = u_1 + u_2 = u_{k1} + u_{k2} \)

\( 0.3x + 0 = 0.3x + 0 + 0 \times V_2 \)

(b) \[ 0.3 \times 2 + 0 = (3 + 3) \times 12 \]

\( 3x + 2u + 0 \Rightarrow \) Force of friction on \( P \) before collision

Horizontal plane \( F = 4 \times 0.3g \)

\( 4 \times 0.3g \times 15 = \frac{1}{2} \times 2 \times 15 \times u \)

\( u = 0.7 \)

Coefficient of friction \( \mu = 0.7 \)
7 (a) \[4 \times 10 + 0 = 4 \times \frac{3}{4} V + 2V \] \[\sqrt{\text{Answers}}\]
\[\Rightarrow V_A = 5 \text{ and } V_B = 10 \text{ ms}^{-1}\]

(b) for B and C
\[2 \times 10 + 0 = 2 \times V_B + 1 \times V_C\]
\[20 = 2 \times \frac{1}{3} V_C + V_C\]
\[\Rightarrow V_C = 12 \quad (V_B = \frac{1}{3} V_C)\]
\[\Rightarrow V_B = \frac{1}{3} V_C = \frac{1}{3} \times 12 = 4\]
\[V_B = 4 \quad \text{and} \quad V_A = 5 \quad \Rightarrow V_A > V_B\]

(c) for A and B
\[4 \times 5 + 2 \times 4 = 4V + 3V\]
\[\Rightarrow V = 14 \frac{1}{3} \text{ ms}^{-1}\]

Initial K.E. = \[\frac{1}{2} \times 4 \times 10^2\]
K.E. Final = \[\frac{1}{2} \times (4 + 2) \times \left(14\frac{1}{3}\right)^2\]
\[+ \frac{1}{2} \times 1 \times 12^2\]
\[\text{K.E. initial} = 200 - \frac{472}{3}\]
\[= 188\frac{2}{3}\]

8. Using conservation of
momentum;
\[m \times 2 + 0 = m (-0.5) + 0.2 \times 1\]
\[\Rightarrow m = 0.08\]