Mechanics 2

Equilibrium of a Rigid Body
Exercise.

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1. A child's toy consists of a uniform solid circular cone, of vertical height 2r and radius r, and a uniform solid hemisphere of radius r. The circular base of the cone and the hemisphere are joined together so that they coincide. The cone and the hemisphere are made of the same material. Show that the centre of mass of the toy is at a distance \( \frac{27}{5} r \) from the vertex of the cone.

2. A uniform solid cone has weight 5N and base radius 0.1m. AB is a diameter of the base of the cone. The cone is held in equilibrium, with A in contact with a rough horizontal surface and AB vertical, by a force applied at B. This force has magnitude 3N and acts parallel to the axis of the cone. Calculate the height of the cone.

3. ABC is the cross-section through the centre of mass of a uniform prism which rests with AB on a rough horizontal surface. AB = 0.4m and C is 0.9m above the surface. The prism is on the point of toppling about its edge through B. Show that angle BAC = 43.4° correct to 3 significant figures.

   (i) A force of magnitude 18N acting in the plane of the cross-section and perpendicular to AC is now applied to the prism at C. The prism is on the point of rotating about its edge through A.

   (ii) Calculate the weight of the prism.

   (iii) Given also that the prism is on the point of slipping, calculate the coefficient of friction between the prism and the surface.
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4. ABCD is a uniform lamina in the shape of a trapezium which has
   centre of mass G. The sides AD and
   BC are parallel and 1.8 m apart,
   with AD = 2.4 m and BC = 1.2 m.
   (i) Show that the distance of G
       from AD is 0.8 m. -- [4]

   The lamina is freely suspended at A and hangs in
   equilibrium with AD making an angle of 30° with the vertical.
   (ii) Calculate the distance AG. -- [2]

   with the lamina still freely suspended at A a horizontal
   force of magnitude 7N acting in the plane of the lamina is applied
   at D. The lamina is in equilibrium with AG making angle of 10°
   with the horizontal and vertical.
   (iii) Find the two possible values for the weight of the lamina. -- [5]

5. The diagram shows the cross-
   section through the centre of mass
   of a uniform solid object. The
   object is a cylinder of radius 0.2m
   and length 0.7m, from which a
   hemisphere of radius 0.2m has been removed at one end.
   The point A is the centre of the plane face at the other end of the
   object. Find the distance of the centre of mass of object from A. -- [5]

6. ABC is a uniform lamina in the
   form of a triangle with AB = 0.3m, 0.3m
   BC = 0.6m and a right angle at B.
   (i) State the distance of the centre of mass
       of the lamina from AB and from BC.
       Distance from AB ---
       Distance from BC ---
       (Continued->)
6. The lamina is freely suspended at B and hangs in equilibrium.
   (ii) Find the angle between AB and the horizontal. --[3]

   A force of magnitude 12 N is applied along the edge AC of the lamina in the direction from A towards C. The lamina, still suspended at B, is now in equilibrium with AB vertical.

   (iii) Calculate the weight of the lamina. --[3]

7. A uniform lamina ABCDEFG is formed from a square ABDG by removing a smaller square CDFF from one corner.

   AB = 0.7m and DF = 0.3m. Find the distance of the centre of mass of the lamina from A. --[4] S-19 52 43

8. Fig. 1 shows an object made from a uniform wire of length 0.8m. The object consists of a straight part AB, and a semicircular part BC such that A, B, and C lie in the same straight line. The radius of the semicircle is 0.4m and the centre of mass of the object is 0.1m from line ABC.

   (i) Show that \( z = 0.2 \) --[3]

   The object is freely suspended at A and a horizontal force of magnitude 7N is applied to the object at C so that the object is in equilibrium with ABC vertical. (Fig 2)

   (ii) Calculate the weight of the object. --[3] S-19 52 47

   The 7N force is removed and the object hangs in equilibrium with ABC at an angle of 8° with the vertical.

   (iii) Find \( \theta \). --[3] S-19 52 47
9. A uniform object is made by joining together the solid cubes with edges 3m, 2m and 1m. The object has an axis of symmetry, with the cubes stacked vertically and the cube of edge 2m between the other two cubes.

(i) Calculate the distance of the centre of mass of the object above the base of the largest cube.

The smallest cube is now removed from the object. It is replaced by a heavier uniform cube with 1m edges which is made of a different material. The centre of mass is now at the base of 2m cube.

(ii) Find the ratio of the masses of the two cubes of edge 1m.

10. Fig.1 shows the cross-section of a cylinder through which a cylindrical hole has been drilled to make a uniform prism. The radius of the cylinder is $r$, and the radius of the hole is $r$. The centre of the hole is at a distance of $2r$ from the centre of the cylinder.

(i) Find in terms of $r$, the distance of the centre of mass of the prism from the centre of the cylinder.

The prism has weight $w$ and is placed with its curved surface on a rough horizontal plane. The axis of symmetry of the cross-section makes an angle of $30^\circ$ with the vertical. A horizontal force of magnitude $P$ is acting in the plane of cross-section through the centre of mass is applied to the cylinder at the highest point of this cross-section.

The prism rests in limiting equilibrium. (continued)
10(ii) Find the coefficient of friction between the prism and the plane.

11. A uniform rectangular block has a square base ABCD with \( AB = BC = 0.4 \text{ m} \). The height of the block is \( h \text{ m} \). The block is placed with its base on a rough plane inclined at 30° to the horizontal. The block does not slide. It is given that the block is on the point of toppling when the diagonal \( AC \) lies along a line of greatest slope. Calculate \( h \). --- [3]

12. ABCD is a uniform square lamina with sides of length 0.6 m. A circular hole of radius 0.3 m is made in the lamina. The centre of the hole is 0.3 m from AB and 0.25 m from AD. The lamina is freely suspended at A and hangs with the axis of symmetry making an angle of 45° with the horizontal.

(i) Show that \( r = 0.214 \), correct to 3 significant figures.

The lamina is held in equilibrium with AD horizontal by a force of magnitude 15 N acting in the plane of the lamina applied at D. The line of action of this force makes an angle of 60° with the vertical.

(ii) Find the weight of the original lamina, before the hole was made. --- [14]
13. A non-uniform rod AB of length 0.5 m and weight 8 N is freely hinged to a fixed point at A. The rod makes an angle of 30° with the horizontal with B above the level of A. The rod is held in equilibrium by a force of magnitude 12 N acting in the vertical plane containing the rod at an angle of 30° to AB applied at B. Find the distance of the centre of mass of the rod from A. \[ S-18/51/2 \] \[ \[3\] \]

14. A uniform solid cone has height 1.2 m and base radius 0.5 m. A uniform object is made by drilling a cylindrical hole of radius 0.2 m through the cone along the axis of symmetry.

(i) Show that the height of the object is 0.72 m and that the volume of the cone removed by the drilling is 0.0352 m³. \[ S-18/51/2 \] \[ \[4\] \]

(ii) Find the distance of the centre of mass of the object from its base. \[ S-18/51/2 \] \[ \[6\] \]

15. ABC is an object made from a uniform wire consisting of two straight portions AB and BC, in which AB = \( a \), \( BC = x \) and angle ABC = 90°. When the object is freely suspended from A and is in equilibrium, the angle between AB and the horizontal is \( \theta \).

(i) Show that \( x^2 \tan \theta - 2ax - a^2 = 0 \) \[ S-18/52/3 \] \[ \[3\] \]

(ii) Given that \( \tan \theta = 1.25 \), calculate the length of the wire in terms of \( a \). \[ S-18/52/3 \] \[ \[2\] \]
16. A uniform object is made by joining a solid cone of height 0.8 m and base radius 0.6 m and a cylinder. The cylinder has length 0.4 m and radius 0.5 m. A cylindrical hole of length 0.4 m and radius 2 m drilled through it along the axis of symmetry. A plane face of the cylinder is attached to the base of the cone so that the object has an axis of symmetry perpendicular to its base and passing through the vertex of the cone. The object is placed with points on the base of the cone and the base of the cylinder in contact with a horizontal surface. The object is on the point of toppling.
(i) Show that the centre of mass of the object is 0.15 m from the base of the cone.
(ii) Find x.

17. A uniform object is made by attaching the base of a solid hemisphere to the base of a solid cone so that the object has an axis of symmetry. The base of the cone has radius 0.3 m, and the hemisphere has radius 0.3 m. The object is placed on a horizontal plane with a point A on the curved surface of the hemisphere and a point B on the circumference of the cone in contact with the plane.
(i) Given that the object is on the point of toppling about B, find the distance of the centre of mass of the object from the base of cone.
(ii) Given instead that the object is on the point of toppling about A, calculate the height of the cone.
Q 18. Fig 1. shows the cross-section ABCDE through the centre of mass G of a uniform prism. The cross-section consists of a rectangle ABCF from which a triangle DEF has been removed; AB = 0.6 m, BC = 0.7 m and DF = EF = 0.3 m.

(i) Show that the distance of G from BC is 0.27 m and find the distance of G from AB. 

The prism is placed with CD on a rough horizontal surface. A force of magnitude 2 N acting in the plane of cross-section is applied to the prism. The line of action of the force passes through G and is perpendicular to DE. The prism is at the point of toppling about the edge through D.

(ii) Calculate the weight of the prism. [W-18/51/86] -- [3]

19. A uniform solid object is made by attaching a cone to a cylinder so that the circumferences of the base of the cone and a plane face of the cylinder coincide. The cone and the cylinder each have radius 0.3 m and height 0.4 m.

(i) Calculate the distance of the centre of mass of the object from the vertex of the cone. [W-18/52/63] -- [4]

The object has weight 2 N and is placed with its plane circular face on a rough horizontal surface. A force of magnitude k N acting at 30° to upward vertical is applied to the vertex of the cone. The object does not slip.

(ii) Find the greatest possible value of k for which the object does not topple. [W-18/51/86] -- [3]
20. The diagram shows a uniform lamina ABCDEFGH. The lamina consists of a quarter-circle OAB of radius r m, a rectangle DEFG and two isosceles right-angled triangles COD and GDH. The rectangle has DG = EF = x m and DE = FG = x m.

(i) Given that the centre of mass of the lamina is at O, express x in terms of r. --- [6]

(ii) Given instead that the rectangle DEFG is a square with edges of length x m, state with a reason whether the centre of mass of the lamina lies within the square or the quarter-circle. [M-18/52/Q6] --- [1]

21. A cylindrical container is open at the top. The curved surface and the circular base of the container are both made from the same thin uniform material. The container has radius 0.2 m and height 0.9 m.

(i) Show that the centre of mass of the container is 0.405 m from the base. --- [3]

The container is placed with its base on a rough inclined plane. The container is in equilibrium on the point of slipping down the plane and also on the point of tipping.

(ii) Find the coefficient of friction between the container and the plane. [M-17/52/Q2] --- [3]

22. The diagram shows a uniform lamina ABCD with AB = 0.75 m, AD = 0.6 m, and BC = 0.9 m.

(i) Show that the distance of the centre of mass of the lamina from AB is 0.38 m and find the distance of the centre of mass from BC. --- [5]

The lamina is freely suspended at B and hangs in equilibrium.

23. An object is made from a uniform solid hemisphere of radius 0.56 m and centre O, by removing a hemisphere of radius 0.38 m and centre O. The diagram shows a cross-section through O of the object.

(i) Calculate the distance of the centre of mass of the object from O. [4]

The object has weight 24 N. A uniform hemisphere H of 0.28 m is placed in the hollow part of the object to create a non-uniform hemisphere with centre O. The centre of mass of the non-uniform hemisphere is 0.15 m from O.

(ii) Calculate the weight of H. [5 17/51 8.3] [3]

24. A semicircular lamina of radius 0.7 m and weight 14 N has diameter AB. The lamina is in a vertical plane with A freely pivoted at a fixed point. The straight edge AB rests against a small smooth peg P above the level of A. The angle between AB and the horizontal is 30° and AP = 0.9 m.

(i) Show that the magnitude of the force exerted by the peg on the lamina is 7.13 N, correct to 3 significant figures. [14]

(ii) Find the angle with the horizontal of the force exerted by the pivot on the lamina at A. [5 17/51 8.5] [3]

25. An open box in the shape of a cube with edges of length 0.3 m is placed with its base horizontal and its four sides vertical. The four sides and base are uniform laminas, each with weight 3N.

(i) Calculate the height of the centre of mass of the box above its base. [5]

The box is now fitted with a thin uniform square lid of weight 3N and with edges of length 0.2 m. The lid is attached to the box by a hinge of length 0.2 m and weight 2N. The lid of the box is kept partly open. (Continued→)
25/11 (continued)

Find the angle which the lid makes with the horizontal when the centre of mass of the box (including the lid and hinges) is 0.12m above the base of the box.  ---[4]

26. The end A of a non-uniform rod AB of length 0.6m and weight 8N rests on a rough horizontal plane, with AB inclined at 60° to the horizontal. Equilibrium is maintained by a force of magnitude 3N applied to the rod at B.

This force acts at 30° above the horizontal in the vertical plane containing the rod.

(i) Find the distance of the centre of mass of the rod from A. ---[2]

The 3N force is removed, and the rod is held in equilibrium by a force of magnitude PN applied at B, acting in the vertical plane containing the rod, at an angle of 30° below the horizontal.

(ii) Calculate PN. ---[2]

In one of the two situations described, the rod AB is in limiting equilibrium.

(iii) Find the coefficient of friction at A. [5.17 52 06] ---[4]

34. A solid object consists of a uniform hemisphere of radius 0.4m attached to a uniform cylinder of radius 0.4m so that the circumferences of their circular faces coincide. The hemisphere and cylinder each have weight 20N. The centre of mass of the object lies at the centre O of their common circular face.

(i) Show that the height of the cylinder is 0.3m. ---[3]

A new object is made by cutting the cylinder in half and removing the half not attached to the hemisphere. The cut is perpendicular to the axis of symmetry, so the new object consists of a hemisphere and a cylinder half the height of the original cylinder.

(ii) Find the distance of the centre of mass of the new object from O. ---[4]
27. The new object is placed with its hemispherical part on a rough horizontal surface. The new object is held in equilibrium by a force of magnitude PN acting along its axis of symmetry, which is inclined at 30° to the horizontal.

(iii) Find P.

29. A uniform solid cone has height 0.6 m and base radius 0.3 m. A uniform hollow cylinder, open at both ends, has the same dimensions. An object is made by filling the cone inside the cylinder so that the base of the cone coincides with one end of the cylinder. The total weight of the object is 60 N and its centre of mass is 0.25 m from the base of the cone. Calculate the weight of the cone.

30. ABC is a uniform lamina in the shape of a quadrant of a circle with centre O and radius 0.8 m, which has its centre of mass at G. The lamina is smoothly hinged at A to a fixed point and is free to rotate in a vertical plane. A horizontal force of magnitude 12 N acting in the plane of the lamina is applied to the lamina at B. The lamina is in equilibrium with AG horizontal.

(i) Calculate the length AG.
(ii) Find the weight of lamina.
31. A uniform solid hemisphere of weight 6N and radius 0.8 m rests in limiting equilibrium with its curved surface on a rough horizontal plane. The axis of symmetry of the hemisphere is inclined at an angle of \( \theta \) to the horizontal, where \( \cos \theta = 0.28 \). Equilibrium is maintained by a horizontal force of magnitude \( PN \) applied at the lowest point of the circular rim of the hemisphere.

(i) Show that \( P = 8.75 \) N

(ii) Find the coefficient of friction between the hemisphere and the plane.

22. A uniform lamina is made by joining a rectangle \( ABCD \), in which \( AB = CD = 0.56 \) m and \( BC = AD = 3 \) m and a square \( EFGA \) of side 1.2 m. The vertex \( E \) of square lies on the edge \( AD \) of the rectangle. The centre of mass of the lamina is at a distance of \( \frac{\text{distance}}{a} \) from \( BC \) and \( \frac{\text{distance}}{b} \) from \( BAG \).

(i) Find the value of \( h \) and show that \( h = \frac{h}{2} \)

The lamina is freely suspended at the point \( B \) and hangs in equilibrium.

(ii) State the angle which the edge \( BC \) makes with the horizontal.

Instead, the lamina is now freely suspended at the point \( F \) and hangs in equilibrium.

(iii) Calculate the angle between \( FB \) and the vertical.
32. A uniform wire has the shape of a semicircular arc, with diameter AB of length 0.8 m. The wire is attached to vertical wall by a smooth hinge at A. The wire is held in equilibrium with AB inclined at 70° to the upward vertical by a light string attached to B. The other end of the string is attached to the point C on the wall 0.8 m vertically above A. The tension in the string is 15 N.

(i) Show that the horizontal distance of the centre of mass of the wire from the wall is 0.463 m, correct to 3 significant figures. [3]

(ii) Calculate the weight of the wire. [5-6/5/182] [2]

34. A uniform solid cone has base radius 0.4 m and height 1.4 m. A uniform solid cylinder has radius 0.4 m and weight equal to the weight of the cone. A object supported by attaching the cylinder to the cone so that the base of the cone and a circular face of the cylinder are in contact and their circumferences coincide. The object rests in equilibrium with its circular base on a plane inclined at an angle of 20° to the horizontal.

(i) Calculate the least possible value of the coefficient of friction between the plane and the object. [2]

(ii) Calculate the greatest possible height of the cylinder. [5-6/5/184] [4]
35. A uniform block is made by drilling a cylindrical hole through a rectangular block. The axis of the cylindrical hole is perpendicular to the cross-section ABCD through the centre of mass of the object.

AB = CD = 0.7 m, BC = AD = 0.4 m, and the centre of the hole is 0.1 m from AB and 0.2 m from AD.

(i) Find the distance of the centre of mass of the object from AB, and calculate the distance of the centre of mass from AD.

(ii) Find the value of F.

(iii) Find the greatest possible value of g.

36. A uniform wire is bent to form an object which has a semicircular arc with diameter AB of length 1.2 m, with a smaller semicircular arc with diameter BC of length 0.6 m. The end C of the smaller arc is at the centre of the larger arc. The two semicircular arcs of the wire are in the same plane.

(i) Show that the distance of the centre of mass of the object from the line ACB is 0.191 m, correct to 4 significant figures.

(ii) Find the angle between ACB and the vertical.
The diagram shows the cross-section ABCD through the centre of mass of a uniform solid prism, AB = 0.9m, BC = 2am, AD = am and angle ABC = angle BAD = 90°.

(i) Calculate the distance of the centre of mass of the prism from AD. -- [3]
(ii) Express the distance of the centre of mass of the prism from AB in terms of a. -- [3]

The prism has weight 18N and rests in equilibrium on a rough horizontal surface, with AD in contact with the surface. A horizontal force of magnitude 5N is applied to the prism. The force acts through the centre of mass in the direction BC.

(iii) Given that the prism is on the point of toppling, calculate a. -- [3]

38. A non-uniform rod AB of length 0.5m is freely hinged to a fixed point A. The rod is in equilibrium at an angle of 30° with the horizontal with B below the level of A.

Equilibrium is maintained by a force of magnitude FN applied at B acting at 45° above the horizontal in the vertical plane containing AB. The force exerted by the hinge on the rod has magnitude 10N and acts at an angle of 60° above the horizontal.

(i) By resolving horizontally and vertically, calculate F and the weight of the rod. -- [4]

(ii) Find the distance of the centre of mass of the rod from A. -- [3]
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39. The diagram shows the cross-section

ABCDE through the centre of mass of a uniform prism which rests with AB on

a rough horizontal ground. ABCD is a rectangle

with AB = CD = 0.4 m and AC = AD = 1.8 m.

The other part of the cross-section is a semicircle

with diameter DE and radius 0.8 m.

(i) Given that the prism is on the point of

topping, show that \( s = 0.6 \). \([-3]\)

A force of magnitude PN is applied to the prism, acting

at 60° to the upwards vertical along a tangent to the semicircle

at a point between D and E. The prism has weight 15 N and is

in equilibrium on the point of toppling about B.

(ii) Show that \( s = 3.26 \), correct to 3 significant figures. \([-4]\)

(iii) Find the smallest possible value of the coefficient of friction

between the prism and the ground. \([-2]\)

40. The diagram shows the cross-section

ABCDDE through the centre of mass of

a uniform prism on a rough inclined

plane. The portion ADEO is a rectangle,

in which \( AD = DE = 0.6 \text{ m} \) and \( AO = AD = 0.8 \text{ m} \)

the portion BCD is an isosceles triangle

in which angle BCD is a right angle, and A is the mid point

of BC. The plane is inclined at 45° to the horizontal, BC lies

along a line of greatest slope of the plane and DE is horizontal.

(i) Calculate the distance the centre of mass of the prism from AD. \([-3]\)

The weight of prism is 21 N, and is held in equilibrium by a

horizontal force of magnitude PN acting along ED.

(ii) Find the smallest value of \( P \) for which the prism does not topple. \([-3]\)

(iii) It is given that the prism is about to slip for this smallest value of \( P \).

Calculate the coefficient of friction between the prism and the plane. \([-3]\)

(Continued \(\rightarrow\))
40. The value of \( P \) is gradually increased until the prism ceases to be in equilibrium.

(iii) Show that the prism topples before it begins to slide, stating the value of \( P \) at which equilibrium is broken: 

\[ P = \frac{5}{15} \times 51 = 17 \]

41. A triangular frame \( ABC \) consists of two uniform right rods each of length \( 0.8 \text{ m} \) and weight \( 3 \text{ N} \), and a longer uniform rod of weight \( 4 \text{ N} \). The triangular frame has \( AB = BC \), and angle \( BAC = \angle ABC = 30^\circ \).

(i) Calculate the distance of the centre of mass of the frame from \( AB \). 

\[ \text{[Distance]} \]

The vertex \( A \) of the frame is attached to a smooth hinge at a fixed point. The frame is held in equilibrium with AC vertical by a vertical force of magnitude \( FN \) applied to the frame at \( B \).

(ii) Calculate \( F \), and state the magnitude and direction of the force acting on the frame at the hinge.

\[ \text{[Magnitude, Direction]} \]

42. A uniform solid cube with edges of length \( 0.4 \text{ m} \) rests in equilibrium on a rough plane inclined at an angle of \( 30^\circ \) to the horizontal. \( ABCD \) is a cross-section through the centre of the mass of the cube, with \( AB \) along a line of greatest slope. \( B \) lies below the level of \( A \). One end of a light elastic string with modulus of elasticity \( 12 \text{ N/m} \) and natural length \( 0.4 \text{ m} \) is attached at \( C \). The other end of the string is attached to a point below the level of \( B \) on the same line of greatest slope, such that the string makes an angle \( 30^\circ \) with the plane. (Continued→)
42. The cube is on the point of toppling. Find
(i) the tension in the string.
(ii) the weight of the cube.

43. A uniform semicircular lamina has diameter AB of length 0.8 m.
(i) Find the distance of the centre of mass of the lamina from AB.

The lamina rests on a vertical plane, with point B of the lamina in contact with a rough horizontal surface and with A vertically above B. Equilibrium is maintained by a force of magnitude 6 N in the plane of the lamina, applied to the lamina at A and acting at an angle of 20° below the horizontal.

(ii) Calculate the mass of the lamina.

44. A uniform circular disc has centre O and radius 1.2 m. The centre of the disc is at the origin of x-axis and y-axis. Two circular holes with centres at A and B are made in the disc. The point A is on the negative x-axis with OA = 0.5 m, the point B is on the negative y-axis with OB = 0.7 m. The whole with centre A has radius 0.3 m and the hole with centre B has radius 0.4 m. Find the distance of the centre of mass of the object from
(i) the x-axis.
(ii) the y-axis.

The object can rotate freely in a vertical plane about a horizontal axis through O.
(iii) Calculate the angle which OA makes with vertical when the object rests in equilibrium.
An object is formed by joining a hemispherical shell of radius 0.3 m and a solid cone with base radius 0.2 m and height h m along the circumference. The centre of mass, G, of the object is d m from the vertex of the cone on the axis of symmetry of the object. The object rests in equilibrium on a horizontal plane, with the curved surface of the cone in contact with the plane. The object is on the point of toppling.

(i) Show that \( d = h + \frac{0.04}{h} \). \(-3\)

(ii) It is given that the cone is uniform and of weight 4 N, and that the hemispherical shell is uniform and of weight 2 N. Given also that h = 0.8, find W. \( W = 15/53 \)\(^{-6}\)
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1. Density from
   
   Volume = \( \frac{1}{3} \pi r^2 h \)
   
   Com from vertex = \( \frac{3}{4} \pi r^3 \)
   
   Hemisphere = \( \frac{2}{3} \pi r^3 \)
   
   \( \Rightarrow \frac{5}{3} \pi r^3 = \frac{2}{3} \pi r^3 \)

2. Distance from vertex of COM = \( \frac{9}{10} \)

3. Conservation of momentum at B
   
   \( 5 \times \frac{4}{5} = 3 \times 0.2 \)
   
   \( \Rightarrow R = 0.48 \text{ m} \)

4. (i) Rectangle area = \( 1 \times 2 \times 1.8 = 3.16 \)
   
   \( y = 1.5 \times 0.9 \)

   and triangle \( \frac{1}{2} \times 1.3 \times 1.8 = 0.98 \)
   
   \( y = 1.80 - 0.6 = 1.2 \)

   \( (3.16 + 1.08) \times 1.2 = 5.116 \times 0.9 + 1.08 \times 6 \)
   
   \( \Rightarrow y = 0.8 \text{ m} \)

5. (ii) AD makes an angle of 40° with Vertical.

   \( WxG_1 = 7 \times AE \)

   Case II maybe AD makes an angle of 20° with the Vertical.

   Now \( WxG = 7 \times AE \)

5. \( V \) COM from base

   Cylinder \( \pi \times 0.2^2 \times 0.7 = 0.47 \)

   Hemisphere \( \frac{2}{3} \pi (0.2)^3 = 0.32 \times 0.2 \)

   Let the COM of the object \( \Rightarrow \)

   \( \Rightarrow \bar{x} = 0.285 \)

\[ \text{Scanned with CamScanner} \]
6. (i) From \( AB = 0.2 \) \( \checkmark \)  
From \( BC = 0.1 \) \( \checkmark \)  
(ii) \( \tan \beta = 0.1 \) \( \Rightarrow \beta = 5.6^\circ \)  
\[ \tan \theta = \frac{0.1}{0.3} \]  
\( \Rightarrow \theta = 17.5^\circ \) \( \checkmark \)  
(iii) \( \tan \gamma = \frac{0.1}{0.3} \) \( \Rightarrow \gamma = 17.5^\circ \) \( \checkmark \)  
\[ y = 0.1 \times 10 \times 2 \]  
\[ \Rightarrow y = 2 \]  
\( \Rightarrow b = 0.9 \) \( \checkmark \)  
9. (i) Total Volume \( = (2\pi + 8 + 1) = 3.6 \)  
\[ 36 \times 8 = 2\pi \times 15 + 8 \times 4 + 1 \times 5 \]  
\( \Rightarrow x = 13/6 = 2.17 \) \( \checkmark \)  
(ii) \( \text{Mass of the New Obj} = (35 + m) \)  
\( (35 + m) \times 3 = 2\pi \times 15 + 8 \times 4 + 5 \times 5 \)  
\( \Rightarrow m = 13 \)  
\[ \text{Ratio of both cubes is } 1:13 \text{ (or } 13:1) \]  
10. \( \text{Area of Original Circle} = 35 \pi \times 2^2 \)  
\( \text{Area of Whole} = 17 \pi ^2 \)  
(i) \( \text{Area of Cross Section} = 24 \pi \times 2^2 \)  
\[ 25\pi \times 10 = \pi ^2 \times 2 \times 2 \times 2 + 24 \pi \times 1 \times d \]  
\( \Rightarrow d = \sqrt{2} = 0.08333 \) \( \checkmark \)  
(ii) \( P (2 \times 5 \times 2) = W \left( \frac{4}{12} \right) \) \( \Rightarrow 9 \)  
\( \Rightarrow P = W \times 160 = W \times 240 = F \)  
\[ u = F = \frac{W}{240} \times 120 = \frac{1}{240} \times 120 = 5 \]  
\[ u = 0.00417 \]  
11. \( \theta = \sqrt{2} \times \frac{1}{2} + 2 \times \frac{1}{2} \)  
\( \Rightarrow \lambda = 0.2 \) \( \checkmark \)  
(ii) \( AC = AB + BC \)  
\[ AC = (0.8 - 0.2) + 2 \times 0.2 \]  
\( = 0.5 \times 168 \)  
\( W \times 0.1 = 7 \times AC \Rightarrow W = 40 N \)  
\[ \Rightarrow W = 0.98 \]  
\[ \Rightarrow 0.98 \]  
\[ \text{Scanned with CamScanner} \]
12. Let M be the centre of circular area A B C D C is the centre of circle O is the centre of circle C in COM

\[ D P = 0.25 \]

\[ P M = 0.03 \]

Let \( PC = \overline{X} \)

\[ \tan 48^\circ = \frac{PC}{PA} = \frac{X}{X-0.25} \Rightarrow X = 0.3333 \]

Now, all distances are taken from point P

(i) Along the line of symmetry PC

\[ 0.08 \times 0.3 = \frac{\pi}{4} \times (X-0.25) \]

\[ \Rightarrow \frac{\pi}{4} \times (X-0.25) = 0.08 \times 0.3 \]

\[ \Rightarrow \frac{\pi}{4} \times (X-0.25) = 0.08 \times (X-0.25) \]

\[ \Rightarrow X = 0.314 \]

(ii) \( 0.3 N = 15 \times 0.3 \times 0.06 \) (At the point of support)

\[ \Rightarrow W = 15 \]

\[ \text{Square} = 15 \times 0.6^2 / (0.6^2 - \pi \times 0.25^2) \]

\[ \Rightarrow \text{Square} = 2.5 \text{ N} \]

13. \( 8 \times 0.30 = 0.5 \times 12 \times \sin 30^\circ \)

\[ \Rightarrow x = 0.4332 \text{ in} \]

14. (i) Height of conical tip \( h = \frac{R}{\sqrt{2}} \times 0.25 \)

\[ \Rightarrow h = 1.2 \times 0.25 \]

\[ \Rightarrow h = 0.48 \]

Height of cylinder = \( 1 - 0.48 = 0.52 \)

Volume of removed

\[ = \frac{1}{3} \pi \times (1.2)^2 \times 0.52 + \pi \times 0.25 \times 0.52 \times 0.72 \]

\[ = 0.006477 \times 0.52 + 0.0288 \times 0.72 \]

\[ = 0.0354 \text{ in}^3 \]

15. (ii) Given \( \alpha = 0.25 \), from (1)

\[ 1.25x^2 - 2ax - a^2 = 0 \]

\[ \Rightarrow \text{Length} = x + a = 2a + a = 3a \]

\[ \text{(continued)} \]
16(i) \[ \tan \theta = \frac{QT}{ST} = \frac{0Q - OT}{ST} = \frac{0.6 - 0.5}{0.4} = \frac{1}{4} \]

\[ \tan \theta = \frac{OG}{0G} \]

\[ OG = \frac{X}{0.6} \]

\[ \frac{OG}{0G} = \frac{X}{0.6} \]

\[ \frac{1}{4} = \frac{X}{0.6} \]

\[ X = 0.15 \text{ m} \]

16(ii) \[ \left( \frac{1}{3} \pi \times 6^2 \times 0.8 \times \frac{8}{4} - \pi (1.5^2 - 1.4^2) \times 0.4 \right) \times \frac{\sqrt{3}}{2} \]

\[ \Rightarrow X = 0.464 \checkmark \]

17(i) \[ S_h \Delta ABC \]

\[ \text{Area} = 0.2 = 0.2 \times 0.3 \]

\[ \Rightarrow \theta = 45.99 \] \[ \tan 48.99 = \frac{X}{0.3} \Rightarrow X = 0.335 \checkmark \]

17(ii) \[ \frac{1}{2} \pi \times (3)^2 \times 0.4 = \frac{8}{3} \pi \times (0.2) \times \frac{3}{2} \times 0.2 \]

\[ h = 0.23 \checkmark \]

18(i) Area of cross-section of 

\[ \text{Area} = 0.2 \times 0.5 - \frac{1}{2} \times 0.3 \times 0.3 = 0.375 \]

\[ 0.375 \times y + 0.42 \times 0.6 - 0.045 (1.6 - 0.3) \]

\[ y = 0.876 \text{ m} \]

Also \[ 0.375 \times 0.42 \times 0.7 - 0.42 \times 0.7 \times 0.3 \]

\[ x = 0.32 \text{ m} \]

18(ii) \[ \frac{FP}{FG} = \frac{4.345}{1.7} \]

\[ \frac{FP}{FG} = 6.15^\circ : 0.7 \]

\[ ED = 0.3 - 0.276 \]

\[ \Rightarrow W = 0.11 \text{ N} \]

19(i) \[ \frac{1}{3} \pi \times (3)^2 \times 4 / 4 = \frac{21}{8} \pi \times (3)^2 \times 4 / 4 \]

\[ \Rightarrow A = 0.44 \]

\[ 0.3 \times 0.3 \times 0.3 \]

\[ 0.3 \times 0.3 \times 0.3 \]

\[ \Rightarrow k = 0.455 \checkmark \]
20(i) Centre of mass of triangle

\[ \text{below } O = \frac{2}{3} \]

Centre of mass of quadrant

\[ \text{below } O = 2.2 \times \sin \frac{\pi}{4} / \frac{3 \times \pi}{4} \]

\[ (x_1)(y_1) = \frac{1}{6}(b^2) + \left( \frac{\pi}{4} \right) \left( \frac{3 \times \pi - \frac{1}{4} \pi}{3} \right) \]

\[ \Rightarrow x = 2 \left[ \frac{2}{3} + \frac{1}{3} \pi \right] \]

\[ \Rightarrow x = 1.01 \times \pi \]

(ii) Within quadrant as the square will be smaller than the rectangle.

23(i) Centre of Mass of hemisphere

\[ \text{hemisphere } = \frac{2 \times 30.56}{8} \text{ or } \frac{3 \times 0.28}{8} \]

\[ \left[ \frac{3 \times \pi}{6} \times 0.56 - \frac{3 \times \pi}{6} \times 0.28 \right] \times = \]

\[ \frac{3}{4} \pi \times 0.56 \times \frac{3}{4} \text{ x } 0.56 \]

\[ - \frac{3}{4} \pi \times 0.28 \times \frac{3}{4} \text{ x } 0.28 \]

\[ \Rightarrow X = 0.285 \text{ m, V} \]

(ii) \( 24 \times 0.285 + W \left( \frac{3}{4} \times 0.28 \right) \)

\[ \Rightarrow W = 40 \text{ N, V} \]

24 COM

\[ \text{OG} = 2.3 \times \sin \frac{\pi}{3} \text{ x } \frac{3}{3} \]

\[ 1 \text{ x } \pi \]

\[ \Rightarrow x = \frac{2 \times 0.7 \times \sin \frac{\pi}{3}}{3 \times \pi} \]

\[ \Rightarrow 0.9 R = 14 \left( 0.7 \times 0.5 - 0.2 \times 0.58 \right) \]

\[ \Rightarrow R = 1.12 \text{ N, V} \]

\[ (ii) H = 7.12, S = 30 \quad \text{and } V = 14 - 7.12 \times 30 \]

\[ \text{tan} \theta = \left( 14 - 7.12 \times 0.3 \right) \]

\[ \frac{7.12 \times \sin 30^\circ}{y} \]

\[ \theta = 65.6^\circ \]

25(i) Height of COM = each vertical line above the base

\[ 5 \times 3 \text{ y } = 4 \times 3 \times 0.1 + 0 \]

\[ y = 0.08 \text{ m, V} \]

\[ \text{(ii) Moment y did about the base} \]

\[ \left( 6 \times 8 + 2 \right) \times 0.13 = 5 \times 8 \times 0.08 + 2 \times 0.2 \]

\[ + 3 \times (0.2 + 0.13 \times 0) \]

\[ \Rightarrow 0 = 4 \text{ in } ^\circ \]

26.5

(i) Area of Base + Area of Triangle

\[ A = 0.5 \times 0.75 + \frac{1}{2} \times 0.2 \times 0.75 \]

\[ A = 0.5625 \]

Now \( 0.5625 \)

\[ X = 7.5 \times 0.6 + 0.8 \]

\[ + \frac{1}{2} \times 0.75 \times 0.75 \times 0.75 \]

\[ \Rightarrow X = 0.39 \text{ m, from 14} \]

\[ y = 0.35 \text{ m, from 0.00} \]

\[ \Rightarrow y = 0.35 \times 0.38 = 0.271 \text{ m, V} \]

\[ \Rightarrow \theta = 42.6^\circ \]
26. (i) \(3 \times 0.6 = 8\times 60\times \overline{x}\)
\[\Rightarrow \overline{x} = 0.45 \text{ m}\]

(ii) \(\frac{AC}{0.6} = \frac{60}{60}\)
\[\Rightarrow \overline{D} = 10\text{ m}\]

(iii) \(\frac{EC}{0.6} = \frac{60}{60}\)
\[\Rightarrow \overline{E} = 10\text{ m}\]

P. \(60\times 0.6 = 8\times 0.45 \times 60\)
\[\Rightarrow P = 6 \text{ N}\]

30(ii) tan BAG = \(0.8435 - 0.9\)
\[\Rightarrow \text{Angle } BAG = 8.5^\circ \text{ or } 8.1^\circ\]
\[\Rightarrow 2.596^\circ = 8.1^\circ\]

W. \(12 \times 2 \times 0.8 \times 60\) 45\(\times\)45 8.1^\circ
\[\Rightarrow 0.572 \times 12 \times 2 \times 0.8 \times 60\]
\[\Rightarrow W = 3.55 \text{ N}\]

31(i) \(60(\frac{7}{3} \times 0.8) \times 0.28 = 9(0.8 - 0.8 \times 0.28)\)
\[\Rightarrow \varphi = 8.75\]

(ii) \(\mu \times 8.75 / 60 = 0.146\)

32(i) \(2 \times 0.56 \times 0.28 + 1.2^2(0.56 + 1.2)\)
\[\Rightarrow \varphi = 0.75\]

Also, \(2 \times 0.56 \times 1 + 1.2^2(1.2)\)
\[\Rightarrow \varphi = 0.75\]

27(i) \(20 \times \frac{3}{8} \times 0.4 = 20 \times \frac{h}{2}\)
\[\Rightarrow h = 0.3 \text{ m}\]

(iii) Cylinder moment = \(10 \times \frac{3}{8} \times 0.15\)
\[\Rightarrow x = 0.075 \text{ m}\]

30(i) \(OG = \overline{x} = 0.075\)
\[\Rightarrow \varphi = 5.625\]

32. (i) \(\text{C.O.M. } (7.75, 7.75), M = 1:\)
\(BM\) will be vertical,
\(\therefore BM\) will be direct and \(BAM\)

(ii) \(\text{C.M. } (7.75, 7.75), M = \text{i}\;
\(BM\) will be vertical,
\(\therefore BM\) will be direct and \(BAM\)

\(\beta = 45^\circ\)

\(\alpha = (0.56 + 1.2 - 0.725) = MR\)
\[1.2 - 0.725\]

\(\beta = 66.7^\circ\)

Here, \(FM\) will be vertical.

\(\therefore FM\) will be vertical.

\(\beta = 66.7^\circ\)

\(\alpha = (0.56 + 1.2 - 0.725) = MR\)
\[1.2 - 0.725\]

\(\beta = 66.7^\circ\)

\[\Rightarrow FR = GF - GR = GF - AQ\]
33. \( d = 6 \times 0.6 + 0.4 \sin 70^\circ \)
   \[ d = 0.463 \text{ m} \]
   \[ d = 0.463 \text{ m} \]

(ii) \( 0.463 \times 15 = 150 \times 0.8 \cos 35^\circ \)
   \[ W = 21.2 \text{ N} \]

34. (i) \[ \mu = \frac{\text{W}}{\text{F}} = \frac{21.2}{320} \]
   \[ \mu = 0.364 \]

(ii) at the height of cylinder = \( x \)

\[ W = \frac{x}{2} + W(x+y) = 24 \times 0.4 \]

\[ O = 0.4 \tan 70 \Rightarrow \mu \text{ of } 0.4 \]

\[ x = 0.732 \]

35. (i) \( 0.7 \times 0.4 \times 0.2 = (0.28 - 0.3)x + 0.03 \times 0.4 \)

\[ x = 0.212 \text{ m} \]

and \( 0.7 \times 0.4 \times 0.35 = (0.28 - 0.03)y + 0.03 \times 0.2 \)

\[ y = 0.368 \]

(ii) \( 0.4 \times F = 0.212 \times 76 \)

\[ F = 37.1 \]

(iii) \( \tan \theta = \frac{0.4 - 0.212}{0.368} \)

\[ (\frac{0.4 - 0.212}{0.368}) \times 76 = 28.6^\circ \]

\[ \theta = 28.6^\circ \]

36. (i) \( \text{COM (large)} = \frac{0.6 \times 0.4 \times 8}{3} \)

\[ \text{COM (large)} = \frac{0.6 \times 0.4 \times 8}{3} \]

\[ \text{COM (large)} = 0.3 \times \frac{0.4 \times 0.3}{2} \]

\[ D = 0.191 \text{ m} \]

37. (i) \( 0.9a + 0.9x = 0.9 \times 0.45 + 2 \times 0.9a \times 0.9 \)

\[ a = 0.75 \]

38. (i) \( F \theta = 45 = 10 \times 0.6 \)

\[ F = 7.1 \]

39. (i) \( \text{COM (semi-circle from DF)} = \frac{28.6 \times 0.4}{3 \pi} \)

\[ \text{COM (semi-circle from DF)} = \frac{28.6 \times 0.4}{3 \pi} \]

\[ (0.4 \times 0.8) \times 0.4 = (\frac{1}{2} \pi^2) \times 4 \times 0.3 \]

\[ \theta = 0.6 \]

(ii) \( P = 10 \times 0.6 + 6 \times 0.6 \times 0.6 \)

\[ P = 10 \times 0.6 + 6 \times 0.6 \times 0.6 \]

\[ P = 3.26 \]

(iii) \( \mu = 3.26 \times 0.60 / (15 - 3.26) \times 0.60 \)

\[ \mu = 0.21 \]
40. (i) \[ d = 0.6 \times 0.8 + 0.6^2 \]
\[ = 0.4(0.6 \times 0.8) - (0.6)^2 \]
\[ \Rightarrow d = 0.43 \text{ m} \checkmark \]
(ii) (a) \[ 21 \times 0.143 = 3.02 \]
\[ \Rightarrow \theta = 2.5 \checkmark \]
(b) \[ F_x = 21 \times 0.45 - 2.5 \times 0.45 \]
\[ R = 21 \times 0.45 + 2.5 \times 0.45 \]
\[ \Rightarrow \mu = \frac{F_x}{R} = 0.787 \checkmark \]
(iii) \[ p \times 0.6 = 21 \times (0.143 + 0.6) \]
\[ \Rightarrow p = 3.6 \]
Required \[ F_x = 2.6 \times 0.45 - 3.1 \times 0.45 \]
Max \[ F_x = 2.6 \times 0.45 \]
As actual \[ F_x < \text{Max } F_x \], no sliding

41. (i) \[ d = (3+3+4) \times 3 \times 0.4 = 3.03 \times 3 \]
\[ \Rightarrow d = 0.12 \text{ m} \]
(ii) \[ (3+3+4) \times 0.12 = F \times 0.8 \times 0.36 \]
\[ \Rightarrow F = 3 \]
at hinge, 7N upwards.

42. (i) \[ CP = 0.8 \]
\[ T = 12(0.8 - 0.4) = 12 \]
\[ \Rightarrow T = 12 \text{ N} \checkmark \]
(ii) Moment \[ T \times b = 0.4 \times 12 \times 60^\circ \]
\[ 0.4 \times 12 \times 60^\circ = 0.8 \times 30^\circ \]
\[ \Rightarrow W = 56.9 \text{ N} \checkmark \]

43. (i) \[ D = \frac{2(0.8)}{3} \times 3.7^\circ \]
\[ \Rightarrow D = 0.17 \text{ m} \checkmark \]
(ii) \[ 0.17 \text{ mg} = 0.8(6 \times 30^\circ) \]
\[ \Rightarrow m = 2.6 \text{ kg} \]

44. (i) \[ \text{Mass of disc} = \pi \left( \frac{1.2^2 - 0.4^2}{4} - \frac{0.3^2}{4} \right) \]
\[ = \pi (0.82 - 0.2 - 0.225) \]
\[ \Rightarrow \theta = 0.094 \checkmark \]
(ii) \[ 0 = \pi (1.2^2 - 0.4^2 - 0.3^2) x - \pi (0.3)^2 x = 0.3 \times 0.5 \]
\[ \Rightarrow x = 0.3 \times 0.5 \checkmark \]
(iii) \[ \mu = \frac{0.094}{0.378} \]
\[ \Rightarrow \theta = 68.1^\circ \]

45. (i) \[ d = \frac{h}{2} = h \]
\[ \Rightarrow d = \frac{h}{2} \checkmark \]

46. (i) \[ d = h + 0.04 \times \vec{AG} \]
\[ \Rightarrow d = h + 0.04 \checkmark \]
(ii) \[ 0.6 \times 4 + 0.9 W = d (4 + W) \]
\[ \Rightarrow d = \frac{0.8 + 0.2^2}{0.8} \]
\[ \Rightarrow 2.4 + 0.9 W = 0.85 (4 + W) \]
\[ \Rightarrow 0.5 W = 1 \]
\[ \Rightarrow W = 20 \checkmark \]

\[ \text{X} \text{ X} \text{ X} \text{ X} \]