Mechanics 2

Equilibrium of a rigid body

Notes

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Equilibrium of a rigid body.

Moments of forces

**Notes**

Moment of a force:

The moment of a force $F$ about a point $O$ is the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the point $O$.

![Diagram](Image)

Fig. 1 (i) Moment of force $F$ about the point $O$ is

$$M = F \times d$$

where $d$ is the perpendicular distance of $O$ from force $F$'s line of action.

![Diagram](Image)

Fig. 2 (ii) Moment of force $F'$ along $\overrightarrow{OP}$, here angle $\angle OPQ = \theta$.

Moment of force about $O' = F' \times d$

$$M = F \times OP \sin \theta$$

- Unit of Moment of force is Nm
- The moment of force creates a turning effect in the rigid body.

**Example 1:** Find the sum of moments about $O$ of the forces shown in the figure.

\[\begin{align*}
\text{Sum} &= -6 \times F_N + 5 \times F_M \\
&= -6 \times 0.819 + 5 \times 1.2 \\
&= 9.1 + 10.9 \\
&= 1.09 Nm \text{ anticlockwise},
\end{align*}\]
Examples: Overhead cables for tramway are supported by uniform, rigid, horizontal beams of weights 1500 N each length the 5 m. Each beam, AB, is freely pivoted at one end A and supports two cables of which may be modelled by vertical loads, each of 1000 N, one 1.5 m from A and the other at 1m from B.

In one situation, the beam is held in equilibrium, resting on a small horizontal support at B, as shown in figure (A).

(i) Draw a diagram showing all the forces acting on beam AB. Show that the vertical force acting on the beam at B is 1850 N.

(ii) In other situations, the beam is supported by a wire, instead of the support at B. The wire is light, attached at one end to the beam at B and at the other to the point C which is 3 m vertically above A, as shown in figure (B).

(iii) Calculate the tension in the wire.

(iv) Find the magnitude and direction of the force on the beam at A.

**Solution:**

Taking moment about A,

\[ R \times 5 = 1000 \times 1.5 + 1500 \times 2.5 + 1000 \times 4 \]

\[ 5R = 9250 \]

\[ R = 1850 \text{ N} \]

Vertical force at B = 1850 N

(Continued →)
(ii) Let the tension in the wire = \( T \) N
Let the wire is inclined at angle \( \theta \)
Angle ABC = 0
Vertical component of \( T \) = \( T \sin \theta \)

\[
T \sin \theta = R
\]

\[
T \times \frac{3}{\sqrt{3^2+5^2}} = 1850
\]

\[
\Rightarrow T = \frac{1850 \times \sqrt{3^2+5^2}}{3}
\]

\[
T = 3595.75
\]

or \( T = 3596 \) N

(iii) Vertical component of force at A = \( F_y \)

\[
F_y + R = 1000 + 1500 + 1650
\]

\[
F_y = 3500 - 1850 = 1650 N \quad (R = 1850 N)
\]

and the horizontal component of \( F = F_x \)

\[
F_x = T \cos \theta
\]

\[
= 3596 \times \frac{5}{\sqrt{3^2+5^2}} = 3083.3 \text{ N}
\]

\[
\therefore F = \sqrt{1650^2 + 3083.3^2} = 3497 \text{ N}
\]

Let the force \( F \) at A is inclined at an angle \( \alpha \) to the horizontal.

\[
\tan \alpha = \frac{F_y}{F_x} = \frac{1650}{3083.3}
\]

\[
\Rightarrow \alpha \approx 28.15^\circ \text{ above the horizontal.}
\]

Force at A = 3497 N

Note: If a body is in equilibrium, the sum of the moments of the forces acting on it, about a point is zero.
Example 3: A uniform beam AB has length 2m and mass 10 kg. The beam is hinged at A and a fixed point on a vertical wall, and is held in a fixed position by a light inextensible string of length 2.4m. One end of the string is attached to the beam at a point 0.7m from A. The other end of the string is attached to the wall at a point vertically above the hinge. The string is at right angle to AB.

(i) Find the tension in the string.

The components of the force exerted by the hinge on the beam are XN horizontally away from the wall and YN vertically downwards.

(ii) Find the values of X and Y.

Solution: Let the beam be inclined to the horizontal at an angle $\theta$.

(i) Angle ACB = $\theta$ = $0.7/21.4$ and $21.4$ and $21.4/3 = 6.5$

Tension in string = $T\sin\theta$ = $100 \cos\theta \times 1 + 260 \times 3 = 680 \times 2.4$

$\Rightarrow T = 960N$.

(ii) $X = T \sin\theta = 960 \times 0.7 = 669N \Rightarrow X = 669N$

Vertically,

And, $Y = 100 + 500 = 700$

$\Rightarrow Y = 700 - 400$

$= 960 \times 2.4 - 400$

$\Rightarrow Y = 921.6 - 400 = 521.6$

$\Rightarrow Y = 522N$.
Equilibrium of a rigid body

Centre of Mass

Notes

Centre of mass:
This is the point at which the whole mass of the body can be considered to be concentrated, or it is the point through which the line of action of the weight of the body always passes, in whatever position the body is placed. (or it is the balance point of a body with size and shape)

Centre of mass of the particles of masses \( m_1, m_2, m_3 \cdots \)
in a straight line, at position \( x_1, x_2, x_3, \cdots \) from one end point \( O \), then \( \bar{x} \) is the distance of the centre of mass from \( O \). Such that:

\[
( m_1 + m_2 + m_3 \cdots ) \bar{x} = x_1 m_1 + x_2 m_2 + x_3 m_3 \cdots
\]

or \( M \bar{x} = \sum m_i x_i \quad \left[ M = \sum m_i \right] \)

Example 4: Find the position of the centre of mass of four particles of masses 1 kg, 4 kg, 3 kg and 2 kg placed on the \( x \)-axis at the points (6,0), (3,0), (2,0) and (4,0) respectively.

Let the 'Centre of mass' \( G \) is at \( (\bar{x},0) \)

Sum of all masses \( M = \sum m_i = 3 + 4 + 2 + 1 = 10 \) kg

\[
\bar{x} = \frac{\sum m_i x_i}{M} = \frac{2 \times 3 + 3 \times 4 + 4 \times 2 + 6 \times 1}{10}
\]

\[
\bar{x} = \frac{32}{10} = 3.2
\]

i.e., The Centre of mass is at \( (3.2,0) \) on \( x \)-axis
Centre of mass for two dimensional bodies:

Let the masses \( m_1, m_2, m_3 \) be placed in a plane, at locations \((x_1, y_1), (x_2, y_2), (x_3, y_3)\).

Then their centre of mass will be at \((x, y)\)

\[
\begin{align*}
x &= \frac{\sum m_i x_i}{\sum m_i} \\
y &= \frac{\sum m_i y_i}{\sum m_i}
\end{align*}
\]

Example 5: A system of three particles consists of 10 kg placed at \((2, 3)\), 15 kg placed at \((4, 2)\) and 25 kg placed at \((6, 6)\). Find the coordinates of centre of mass of the system.

Solution: \(M = \sum m_i = 10 + 15 + 25 = 50\) kg

Let COM \(G\) be at \((x, y)\)

\[
\begin{align*}
x &= \frac{\sum m_i x_i}{\sum m_i} = \frac{10\times 2 + 15\times 4 + 25\times 6}{50} = 4.6 \\
y &= \frac{\sum m_i y_i}{\sum m_i} = \frac{2\times 10 + 15\times 2 + 25\times 6}{50} = 4.2
\end{align*}
\]

Centre of mass \(G\) \((4.6, 4.2)\)

Example 6: A light rectangular metal plate \(PQRS\) has \(PR = 4a\) and \(PS = 2c\), particles of masses \(3k, 5k, 1k\) and \(7k\) are attached respectively to the corners \(P, Q, R\) and \(S\) of the plate. Find the distance of the centre of mass of the loaded plate from

(a) the side \(PQ\) \n(b) the side \(PS\).

(Continued...)
Example 6. Solution:

Rectangular PQRS is placed on the axes such that P is at origin, PA along x-axis and PS along y-axis.

The points P, Q, R and S have coordinates P(0, 0), Q(4, 0), R(4, 2), S(0, 2).

Let the coordinates of Centre of Mass = \((\bar{x}, \bar{y})\)

(a) Distance of Centre of Mass from PA = \(\bar{y}\)

\[
\bar{y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{3 \times 0 + 5 \times 0 + 1 \times 2 + 7 \times 2}{3 + 5 + 1 + 7} = \frac{16}{16} = 1
\]

\(\therefore \bar{y} = 1\)

(b) Distance of Centre of Mass from side PS = \(\bar{x}\)

\[
\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{3 \times 0 + 5 \times 4 + 1 \times 4 + 7 \times 0}{3 + 5 + 1 + 7} = \frac{24}{16} = 1.5
\]

\(\therefore \bar{x} = 1.5\)
Uniform Lamina:

An object whose thickness is very small in comparison to the plane surface (its length and width) is called a uniform lamina and its weight is proportional to the area.

A lamina is uniform if its mass is evenly spread throughout its area.

(i) If a lamina has a axis of symmetry then its then its centre of mass lie on the axis of symmetry.

(ii) If the lamina has more than one axis of symmetry, then the centre of mass lie at the point of intersection of the axes of symmetry.

1. Circular disc. Centre/disc

2. Rectangular lamina, Point of int. $\{l_1, l_2\}$

3. Triangular lamina; Point of int. of medians

$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$$

$$\text{or } AG = \frac{2}{3} AD$$

$$\text{or } GD = \frac{1}{3} AD$$

Here if the coordinates of $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$

$$\text{CDM} - G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
Centre of Mass

Uniform lamina \( \text{COM} \)

4. Sector of a circle\( \frac{2}{3} \) \( \text{rad} \)\( \text{cm} = 0\text{G} \)

5. Semi-circular lamina\( \text{OG} = 2\text{r} \cdot \frac{\text{sw}}{3\pi} \)

Example 7: \( \text{ABC} \) is a uniform lamina in the form of a triangle with \( AB = 0.3 \text{m} \), \( BC = 0.6 \text{m} \) and a right angle at \( B \). State the distance of the centre of mass of the lamina from \( AB \) and \( BC \).

\[ \text{Solution:} \text{let the medians AD and CE intersect at G.} \]
\[ \frac{AD}{GD} = \frac{CG}{GE} = 2 \]
\[ \Rightarrow GE = \frac{1}{3} CE \]
\[ \Rightarrow GQ = \frac{1}{3} BC = \frac{1}{3} \times 0.6 = 0.2 \text{m} \]

The distance of \( \text{COM, G} \) from \( AB = GQ = 0.2 \text{ m} \)

Again \( GP = \frac{1}{2} AB = \frac{1}{2} \times 0.3 = 0.1 \text{ m} \)

Example 8: The diagram shows a uniform semicircular lamina of radius \( \text{cm} \) with centre \( O \).

Solution: \( \text{OG} = 2\text{r} \cdot \frac{\text{sw}}{3\pi} = 6 \text{ cm} \)
Centre of mass of composite lamina:

We can use \( \bar{x} \cdot \Sigma m_i = \Sigma m_i \cdot x_i \)
and \( \bar{y} \cdot \Sigma m_i = \Sigma m_i \cdot y_i \)

Example 9: A uniform lamina ABCD is formed from a square ABCD by removing a smaller square CDEF from one corner. AB = 0.7 m, and DF = 0.3 m. Find the distance of the centre of mass of the lamina from A.

Solution: Draw \( \triangle EHL \),

The given lamina \( ABCD = \text{lamina} \, AHFG + \text{lamina} \, BCDEF \)

\[
\begin{align*}
\text{Area} & : 0.28 \quad 0.12 \quad 0.40 \\
\bar{x} & : 0.35 \quad 0.2 \quad \bar{x} \\
\bar{y} & : 0.2 \quad 0.55 \quad \bar{y}
\end{align*}
\]

\( \bar{x} \cdot \Sigma m_i = \Sigma m_i \cdot x_i \Rightarrow \bar{x} \cdot 0.40 = 0.35 \times 0.28 + 0.12 \times 2 \)
\Rightarrow 0.4 \bar{x} = 0.098 + 0.24 = 0.122
\Rightarrow \bar{x} = \frac{0.122}{0.4} = 0.305 \checkmark

\( \bar{y} \cdot \Sigma m_i = \Sigma m_i \cdot y_i \Rightarrow \bar{y} \times 0.4 = 0.28 \times 0.2 + 0.12 \times 0.55 \)
\Rightarrow 0.4 \bar{y} = 0.056 + 0.066 = 0.122
\Rightarrow \bar{y} = \frac{0.122}{0.4} = 0.305 \checkmark

\( AG = \sqrt{0.305^2 + 0.305^2} = \sqrt{0.18605} \approx 0.431 \checkmark \)

\( \therefore \, AG = 0.431 \checkmark \) (continued →)
Example 9. (Alternate method)

Let us consider uncut square ABCD of side of length 0.7 m.

<table>
<thead>
<tr>
<th>Lamina</th>
<th>Area</th>
<th>COM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Original Sq. ABCD</td>
<td>0.7^2 = 0.49</td>
<td>( A \bar{G}_1 = \frac{0.35^2 + 0.85^2}{0.35 + 0.85} = 0.495 )</td>
</tr>
<tr>
<td>(ii) Small Sq. CDFE</td>
<td>0.3^2 = 0.09</td>
<td>( A \bar{G}_2 = \frac{0.55^2 + 0.55^2}{0.55 + 0.55} = 0.655 )</td>
</tr>
<tr>
<td>(iii) Given lamina</td>
<td>(0.49 - 0.09)</td>
<td>( A \bar{G}_3 = \frac{0.7778}{0.4725} = 0.4 )</td>
</tr>
</tbody>
</table>

Now \( AX \times (0.4) + 0.09 \times 0.7778 = 0.49 \times 0.495 \)
\[ \Rightarrow 0.4 \times AX = 0.49 \times 0.495 - 0.09 \times 0.7778 \]
\[ 0.4 \times AX = 0.24255 - 0.07 = 0.172548 \]
\[ AX = 0.172548 \div 0.4 = 0.43137 \]

\[ \therefore AX = 0.431 \]

Example 10: The diagram shows a uniform lamina ABCD with
AB = 0.75 m and AD = 0.6 m and
BC = 0.9 m, angle BAD = 90°
Show that the distance of the centre of mass of the lamina from AB = 0.38 m and find the distance of the centre of mass from BC.

Solution: Area of Trapezium = Area of Rect + Area of Triangle
\[ A = 0.6 \times 0.75 + 0.3 \times 0.75 \]
\[ \text{Total} \ A = 0.5625 \]

Now \( \bar{x} = \frac{1}{0.5625} \times \left( \frac{0.75 \times 0.6}{2} \right) \times 0.3 \)
\[ + \left( \frac{0.3 \times 0.75}{2} \right) \times (0.6 + 0.3) \]
\[ \Rightarrow \bar{x} = 0.38 \text{ m from } AB \]

Now \( \bar{y} = \frac{1}{0.5625} \times \left( \frac{0.75 \times 0.6}{2} \right) \times 0.375 + \left( \frac{0.3 \times 0.75}{2} \right) \times 0.25 \Rightarrow \bar{y} = 0.35 \)
Example 11: A uniform circular disc has centre O and radius 1.2 m. The centre of the disc is at the origin of the x-axis and y-axis, two circular holes with centres at A and B are made in the disc. The point A is on the negative x-axis with OA = 0.5 m. The point B is on the negative y-axis with OB = 0.7 m. The hole with centre A has radius 0.3 m and the whole with centre B has radius 0.4 m. Find the distance of the centre of mass of the object from:

(i) the x-axis
(ii) the y-axis

Solution:

<table>
<thead>
<tr>
<th>Area</th>
<th>Whole Unhole Circle</th>
<th>Given Lamina 1</th>
<th>Circle (B)</th>
<th>Circle (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>( \pi \times 1.2^2 )</td>
<td>( \pi(1.2^2 - 0.3^2) )</td>
<td>( \pi \times 0.4^2 )</td>
<td>( \pi \times 0.3^2 )</td>
</tr>
<tr>
<td>COM from ( x )-axis</td>
<td>0</td>
<td>0.7 (let)</td>
<td>-0.7</td>
<td>0</td>
</tr>
<tr>
<td>COM from ( y )-axis</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

(i)

Let the centre of mass from \( x \)-axis at distance \( \overline{y} \)

\[
(\pi \times 1.2^2) \times 0 = \pi(1.2^2 - 0.4^2 - 0.3^2) \times \overline{y} + (\pi \times 0.4^2) \times (-0.7) \\
+ (\pi \times 0.3^2) \times 0
\]

\[\Rightarrow 0 = 1.19 \overline{y} - 0.112 \Rightarrow \overline{y} = 0.0941 \text{ m} \checkmark \]

Distance of Centre of Mass from \( x \)-axis = 0.0941 m \checkmark

(ii)

Again, \( (\pi \times 1.2^2) \times 0 = \pi(1.2^2 - 0.4^2 - 0.3^2) \times \overline{x} + (\pi \times 0.4^2) \times 0 \\
+ (\pi \times 0.3^2) \times (-0.5) \)

\[\Rightarrow 0 = 1.19 \overline{x} - 0.045 \Rightarrow \overline{x} = 0.0378 \checkmark \]

Distance of COM from \( y \)-axis = 0.0378 \checkmark
Centre of mass of a framework made by rods (or wire) using:
Centre of mass of these rods (or wires):

(i) Uniform Circular Arc:

\[ \text{COM is at a distance } \frac{2\alpha}{\sin \alpha} \text{ from } O \]

(ii) A framework made by joining the rods (or wire) using the COM of each rod (or wire).

Example: Find the position of the centre of mass of the framework shown in the diagram which is formed by bending a uniform piece of wire of total length \((12 + 2\pi)\) cm to form a sector of a circle, centre \(O\), radius 6 cm.

Solution: Mass of wire is proportional to its length.

Let \(P\) and \(Q\) are the mid-points of \(OA\) and \(OB\) respectively.

Let \(\text{centre angle } = (2\alpha)\); length of \(\text{arc } AB = (12 + 2\pi) - 12\).

Length of \(\text{arc } AB = 2\alpha \times 6 = 12\alpha = \frac{2\pi}{\text{arc}}

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Length} & \text{Arc } AB & \text{Length of } AB & \text{COM from } O \text{ in } x \text{ direction} & y \text{ from } O \\
\hline
\text{From } 0 \text{ to } A & 6 & 6 & 3 & 3 \times \frac{2\pi}{6} = \frac{2\pi}{2} \times 6 = 3 \times 3 \times 6 = \frac{36\pi}{6} \\
\hline
\text{From } 0 \text{ to } B & 6 & 6 & 3 & 3 \times \frac{2\pi}{6} = \frac{2\pi}{2} \times 6 = 3 \times 3 \times 6 = \frac{36\pi}{6} \\
\hline
\text{Framework } & 12 + 2\pi & 12 + 2\pi & x & y \\
\hline
\end{array}
\]

Along \(x\)-axis, \(\overline{x} = \frac{2\pi}{6} \times 6 \times \frac{2\pi}{2} + \frac{36\pi}{6} \times 6 + \frac{36\pi}{6} \times \frac{6}{2}

\[
\overline{x} (12 + 2\pi) = \frac{36}{6} + \frac{9\pi}{6} = 6 + 3\pi

\Rightarrow \overline{x} = \frac{36 + 18\pi}{12 + 2\pi} = \frac{9(\pi + 2)}{6(1 + \pi)}

\overline{y} (12 + 2\pi) = 0 + 6 \times \frac{1}{2} - 6 \times \frac{1}{2} = 0 \Rightarrow \overline{y} = 0

\text{COM is on the axis of symmetry } \overline{OG} = \frac{9(\pi + 2)}{6(1 + \pi)}
Example 13: A uniform wire is bent to form an object which has a semicircular arc with diameter AB of length 1.2 m, with a smaller arc with diameter BC of length 0.6 m. The end C of the smaller arc is at the centre of the larger arc. The two semicircular arcs of the wire are in the same plane. Show that the distance of the centre of mass of the object from the line ACB is 0.191 m, correct to 3 s.f. 

Solution: Mass of wire is proportional to the length.

\[
\text{COM of Arc} = \frac{\text{Length of Arc}}{\pi}
\]

<table>
<thead>
<tr>
<th>Wire Length</th>
<th>Bigger Circle</th>
<th>Smaller Circle</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1\times0.6)</td>
<td>(\pi\times0.6)</td>
<td>(\pi\times0.3)</td>
<td>((\pi\times0.6 + \pi\times0.3))</td>
</tr>
<tr>
<td>Distance of COM from line ACB</td>
<td>(\frac{0.6\times\pi}{\pi} = \frac{1.2}{\pi})</td>
<td>(\frac{0.3\times\pi}{\pi} = \frac{0.6}{\pi})</td>
<td>(D)</td>
</tr>
</tbody>
</table>

\[
D \times \sum m = 0.6\pi \times \left(\frac{1.2}{\pi} + \frac{0.3\pi}{\pi}\right) \left(-0.6\right)
\]

\[
D \times (0.9\pi) = 0.72 - 0.18
\]

\[
\Rightarrow D = \frac{0.54}{0.9\pi} = 0.191 \text{ m}
\]

\[
\therefore \text{Distance of COM from line ACB} = 0.191 \text{ m}
\]
Centre of Mass
Three dimensional Shapes

1. Cube
   G - Mid point of PQ
   P and Q are the points of intersection of diagonals of the opp. faces of the cube.

2. Cuboid
   G - Mid point of PQ

3. Cylinder (Solid)
   PQ is the line of symmetry joining the centres of the circular bases.
   G - Mid point of PQ
   \[ GQ = \frac{h}{2} \]

4. Solid Cone or Pyramid
   \[ OG = \frac{3}{4} h \]

5. Solid Hemisphere
   \[ OG = \frac{3}{8} R \]

6. Hemispherical Shell
   \[ OG = \frac{1}{2} R \]
Example 14: A cylindrical container is open at the top. The curved surface and the circular base of the container are both made from the same thin uniform material. The container has radius 0.2 m and height 0.9 m.

Show that the centre of mass of the container is 0.405 m from the base.

Solution: Mass of the surfaces of cylinder are proportional to the area.
Centre of mass will be on the line of symmetry, joining the centres of circular faces.

<table>
<thead>
<tr>
<th>Area</th>
<th>Curved Surface</th>
<th>Circular Base</th>
<th>Total Container</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\pi r h$</td>
<td>$\pi r^2 = \pi (0.2)^2$</td>
<td></td>
<td>$(2\pi \times 0.2 \times 0.9) + \pi (0.2)^2$</td>
</tr>
<tr>
<td>$= 2\pi \times 0.2 \times 0.9$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COM along the line of symmetry above O_1: $\frac{0.9}{2} = 0$  

$\bar{y} = g \sum m_i y_i$  

$= \int (2\pi \times 0.9 \times 0.2 + \pi (0.2)^2) = (2\pi \times 0.2 \times 0.9) \times 0.9 + \pi (0.2)^2 \times 0.9$  

$= \frac{0.162 \pi}{0.4 \pi} = 0.405$

\[ \bar{y} = \frac{0.405}{\pi} \]

Centre of mass is along the line of symmetry at distance $0.405$ m from base.
Example 15: An object is made from a uniform solid hemisphere of radius 0.56 m and centre O, by removing a hemisphere of radius 0.28 m and centre O.

(i) Calculate the distance of the centre of mass of the object from O. [4]

The object has weight 24 N. A uniform hemisphere of radius 0.28 m is placed in the hollow part of the object to create a non-uniform hemisphere with centre O. The centre of mass of the non-uniform hemisphere is 0.15 m from O.

(ii) Calculate the weight of H. [5]

Solution: Centre of mass of solid hemisphere = \( \frac{3}{8} \) r from O.

<table>
<thead>
<tr>
<th>Volume</th>
<th>Full hemisphere of r = 0.56</th>
<th>Small hemisphere r = 0.28</th>
<th>The Object</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{2}{3} \pi (0.56)^3 )</td>
<td>( \frac{2}{3} \pi (0.28)^3 )</td>
<td>( \frac{2}{3} \pi (0.56^3 - 0.28^3) )</td>
</tr>
</tbody>
</table>

\( d \cdot \Sigma m_i = \Sigma m_i \cdot d_i \)

\[
0.21 \times \frac{2}{3} \pi (0.56)^3 = 0.105 \times \frac{2}{3} \pi (0.28)^3 + \frac{2}{3} \pi (0.56^3 - 0.28^3) \times d
\]

\[
\Rightarrow 0.21 \times 175616 = 0.105 \times 5.2495 + 0.15 \times 3664 \times d
\]

\[
\Rightarrow 0.03688 = 0.0023 + 0.015 \times 3664 \times d
\]

\[
\Rightarrow d = \frac{0.03688 - 0.0023}{0.15} = 0.015 \times 3664 = 0.285 \text{ m}
\]

Again, New total solid hemisphere previous object

\[
(24 + W) \times 0.15 = 24 \times 0.225 + W \left( \frac{3}{8} \times 0.28 \right)
\]

\[
\Rightarrow W = 40 \text{ N}
\]
Example 16: A child's toy consists of a uniform solid circular cone, of vertical height 3R and radius R, and a uniform solid hemisphere of radius R. The circular base of the cone and the hemisphere are joined together so that they coincide. The cone and the hemisphere are made of the same material.

Show that the centre of mass of the toy is at a distance of \( \frac{27}{10} R \) from the vertex of the cone. [FM SP 2013 Q1]

Solution:

\[ \begin{array}{|c|c|c|c|}
\hline
\text{Volume} & \text{Cone} & \text{Hemisphere} & \text{Combined} \\
\hline
\frac{1}{3} \pi R^2 \times 3R & \frac{2}{3} \pi R^3 & \frac{5}{3} \pi R^3 \\
\hline
\text{Distance of COM from vertex of V} & \frac{3R}{4} & 3R + \frac{3R}{8} & \overline{x} \\
\hline
\end{array} \]

\[ \frac{2R}{2} \cdot \Sigma m_i = \Sigma x_i \cdot m_i \]
\[ \frac{5}{3} \pi R^3 \times \overline{x} = \pi R^3 \times \frac{9R}{4} + \frac{3}{8} \pi R^3 \times \frac{9R}{4} \]

\[ \Rightarrow \overline{x} = \frac{27R}{10} \]

Distance of COM from vertex = \( \frac{27R}{10} \)

Example 17: The diagram shows the cross-section through the centre of mass of a uniform solid object. The object is a cylinder of radius 0.2m and height 0.7m, from which a hemisphere of radius 0.2m has been removed at one end. The point A is the centre of the plane face at the other end of the object. Find the distance of the centre of mass of the object from A. [5-19 57 Q3]

Solution:

\[ \begin{array}{|c|c|c|}
\hline
\text{Object} & \text{Uncut Cylinder} & \text{Hemisphere} \\
\hline
\text{Volume} & \pi \times 0.2^2 \times 0.7 & \frac{2}{3} \pi \times 0.2^3 \\
\hline
\text{x-COM from A} & 0.2 & (0.7 - \frac{3}{8} \times 0.2) \\
\hline
\end{array} \]

\[ \overline{x} = \overline{x}_1 = \overline{x}_2 \]
\[ \Rightarrow \frac{0.2}{2} \times \pi \times 0.2^2 \times 0.7 + \frac{1}{2} \left[ \frac{\pi \times 0.2^2 \times 0.7}{3} - \frac{3}{8} \pi \times 0.2^3 \right] \]

\[ \Rightarrow \overline{x} = 0.285 \]
Equilibrium of a rigid body

Equilibrium of a lamina when suspended freely from a fixed point (or pivoted freely about a horizontal axis) will rest in equilibrium in a vertical plane with its centre of mass vertically below the point of suspension (or the pivot).

Example 18: OAB is a uniform lamina in the shape of a quadrant of a circle with centre O and radius 0.8 m, which has its centre of mass at G. The lamina is smoothly hinged at A to a fixed point and is free to rotate in the vertical plane. A horizontal force of magnitude 12 N acting in the plane of the lamina is applied to the lamina at B. The lamina is in equilibrium with A as horizontal. (i) Calculate the length of AG. 

Solution: G lies on the line of symmetry of the sector of circle, angle AOG = \( \frac{\pi}{4} \) radians.

(i) OG produced, \( \text{OP} \bot \text{AB} \)

In \( \triangle AOG \):
\[ \text{OP} = 0.8 \text{ m}, \quad \text{and} \quad \text{AP} = 0.8 \times \cos \left( \frac{\pi}{4} \right) \]

\[ \text{Centre of Mass, } OG = \frac{2}{3} \times 0.8 = 0.533 \text{ m} \]

\[ \text{Now in } \triangle AOG, \]
\[ \text{AG}^2 = \text{GP}^2 + \text{AP}^2 \]
\[ = \left( \text{OP} - 0.8 \right)^2 + \left( 0.8 \times \cos \left( \frac{\pi}{4} \right) \right)^2 \]
\[ = \left( 0.8 \cos \frac{\pi}{4} - 0.48 \right)^2 \]
\[ \Rightarrow \text{AG} = 0.572 \text{ m} \quad \text{(1)} \]

(ii) \( \text{det angle } \angle GAP = \theta = \angle \Delta ABC \)
\[ \tan \theta = \frac{\text{GL}}{\text{AP}} = \frac{\text{OP} - 0.8}{\text{AP}} \]
\[ \Rightarrow \theta = 8.6^\circ \]

Now taking moment about A,
\[ W \times AG = 12 \times AC \]
\[ W \times 0.572 = 12 \times 2 \times 0.8 \times \sin 8.6^\circ \]
\[ W = 12 \times \frac{0.8 \times 0.48}{0.8 \times 0.572} \]
\[ W = 3.55 \text{ N} \]
Example 19: ABC is a uniform lamina in the form of a triangle with \( AB = 0.3 \text{ m} \), \( BC = 0.6 \text{ m} \) and a right angle at B.

(i) State the distance of the centre of mass of the lamina from AB and from BC.

The lamina is freely suspended at B and hangs in equilibrium.

(ii) Find the angle between AB and the horizontal.

A force of magnitude 12 N is applied along the edge AC of the lamina in the direction from A towards C. The lamina is still suspended at B and is now in equilibrium with AB vertical.

(iii) Calculate the weight of the lamina.

Solution: Let the median's AD and BE

(i) intersect at G; G is the COM.

Draw \( GP \perp AB \), \( G \perp BC \).

Distance of \( G \) from \( AB = GP = \frac{1}{2} \times 0.6 = 0.3 \text{ m} \)

and the distance of \( G \) from \( BC = GQ = \frac{1}{2} \times 0.9 = 0.45 \text{ m} \)

(ii) When lamina is freely suspended from \( B \), \( BG \) is vertical. Let \( AB \) be inclined at angle \( \theta \) with horizontal \( BX \).

\[ \angle BGE = \theta \Rightarrow \tan \theta = \frac{GP}{GQ} = \frac{0.3}{0.45} = \frac{2}{3} \]

\[ \theta = \tan^{-1} \left( \frac{2}{3} \right) \approx 33.7^\circ \]

(iii) Let the weight of the lamina \( W \) passes through \( G \), and \( GW \) is vertical.

Draw \( BR \perp AC \), angle \( ABR = 26.6^\circ \).

Now taking moment about point B.

\[ W \times BR = 12 \times BR \]
\[ W \times 0.2 = 12 \times 0.3 \times 0.8 \]
\[ W = 16.1 \text{ N} \]
Example 3. A uniform lamina of weight 15 N has dimensions as shown in the diagram.

(i) Show that the distance of the centre of mass of the lamina from AB is 0.32 m.

The lamina is freely hinged at B to a fixed point. One end of a light inextensible string is attached to the lamina at C. The string passes over a fixed smooth pulley and a particle of mass 1.1 kg is attached to the other end of the string. The lamina is in equilibrium with BC horizontal. The string is taut and makes an angle of 0° with the horizontal at C, and the particle hangs freely below the pulley.

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(ii) Find the value of θ.

Solution: Draw PA ⊥ AB, we have two rectangles BCRA and PQAD

Let the COM of object from AB = \( \frac{\overline{AB}}{2} \)

\[ \overline{\text{G}} \times (0.6 \times 0.3 + 0.4 \times 0.2) = (0.6 \times 0.3) \times 0.3 + (0.4 \times 0.2) \times 0.1 \]

\[ = 0.03 \times 0.09 + 0.008 \]

\[ \overline{\text{G}} = 0.04 \]

\[ \overline{\text{G}} = 0.23 \text{ m} \]

Now det G in the COM

W = 15 N acts at G, GM AB

GM x = 0.23 m

Now taking moment about the point B:

\[ W \times BF = T \times BS \]

\[ 15 \times 0.23 = 11 \times 0.6 \times \sin 0 \]

\[ BS = 0.6 \sin 0 \]

\[ BF = GM = 0.23 \]

\[ \sin 0 = 15 \times 0.23 = 0.5 \]

\[ 0.6 \times 11 \]

\[ 0 = 0.5 \times 0.5 \]

\[ = 30° \]
Solution 21: Let G be the centre of mass.

\[ \angle AOB = \frac{\pi}{4}, \text{ line of symmetry} \]
\[ \text{bisect } \angle AOC, \quad \angle AOS = \frac{\pi}{4} \]
\[ OA = OB = S = 1.5, \quad OM = AB \]
\[ AB = \sqrt{1.5^2 + 1.5^2} = \sqrt{4.5} \approx 2.1213 \]
\[ \Rightarrow AM = AB/2 = 2.1213/2 = 1.0606 \quad \text{(1)} \]

Now from \( CDM \rightarrow G \) \[ OG = \frac{8.88 \times 1.58 \pi}{4} \]
\[ \Rightarrow OG = \frac{6 \times 1}{\sqrt{2}} = 1.3504 \]
\[ \therefore OG = 1.35 \checkmark \]

Now \[ MG = OM - OG = 1.35 - 1.0606 = 0.2898 \]
\[ MG = 0.2898 \checkmark \]

In \( \Delta AMG \), \( \tan \theta = \frac{MG}{OM} = \frac{0.2898}{1.0606} \]
\[ \theta = 15.28^\circ \Rightarrow \theta = 15.3^\circ \checkmark \]

Example 21: A uniform rigid wire AB is in the form a circular arc of radius 1.5 m with centre O. The angle AOB is a right angle. The wire is in equilibrium, freely suspended from the end A. The chord AB makes an angle of \( \theta \) with the vertical.

(i) Show that the distance of the centre of mass of the arc from O is 1.35 m, correct to 3 s.f.

(ii) Find the value of \( \theta \).
Example 22: A non-uniform rod AB of length 0.5 m and weight 8 N is freely hinged to a fixed point at A. The rod makes an angle of 30° with the horizontal, with B above the level of A. The rod is held in equilibrium by a force of magnitude 12 N acting in the vertical plane containing the rod at an angle of 30° to AB, applied at B. Find the distance of the centre of mass of the rod from A.

Solution: Let the centre of mass of rod be C and OG = x m. Draw AC and GC.

The weight of rod vertically at C.

The component of force 12 N perpendicular to AB = 12 \times \sin 30°

Now taking moment about the point A.

\[ W \times AC = AB \times 12 \times \sin 30° \]

\[ 8 \times x \times 60 \times 30° = 12 \times \sin 30° \times 0.5 \]

\[ x = \frac{12 \times 60 \times 30° \times 0.5}{8 \times 60 \times 30°} = 0.433 \]

\[ x = 2 \times 60 \times 30° \]

\[ \therefore \text{COM is at a distance of 0.433 m from A.} \]
Example 23: ABC is an object made from a uniform wire consisting of two straight portions AB and BC, in which $AB = a$, $BC = x$ and angle $ABC = 90^\circ$. When the object is freely suspended from A and is in equilibrium, the angle between AB and the horizontal is $\theta$.

(i) Show that $x^2 \tan \theta - 2ax - a^2 = 0$  
(ii) Given that $\tan \theta = 1.25$, calculate the length of the wire in terms of $a$.

Solution: Mass is proportional to the length.

mass of rod $AB = a$, mass of rod $BC = x$

(i) Taking moment about A,

\[ x \cdot AS = a \cdot AE \quad \text{(1)} \]

\[ AS = 5R - AR = 2R - AR \]

\[ = \frac{a}{2} \sin \theta - a \cos \theta \quad \text{(2)} \]

and \[ AE = \frac{a}{2} \cos \theta \quad \text{(3)} \]

from (2) and (3) in (1)

\[ x \left( \frac{a}{2} \sin \theta - a \cos \theta \right) = a \times \frac{a}{2} \cos \theta \]

\[ \Rightarrow \frac{a^2}{2} \sin \theta = a \cos \theta \left( a + \frac{a}{2} \right) \]

\[ \Rightarrow \frac{a^2}{2} \tan \theta = a + \frac{a}{2} \]

\[ \Rightarrow x^2 \tan \theta - 2ax - a^2 = 0 \quad \text{(4)} \]

Given $\tan \theta = 1.25$

from (4) \[ 1.25 x^2 - 2ax - a^2 = 0 \]

\[ 5x^2 - 8ax - 4a^2 = 0 \]

\[ (x - 2a)(5x + 2a) = 0 \]

\[ x = 2a \quad \text{or} \quad x = -\frac{2a}{5} \]

Now length of wire $= 2x + a$

\[ = 2a + a \quad (\therefore x = 2a) \]

\[ = 3a \]
Example 24: P is the vertex of a uniform solid cone of mass 5 kg, and O is the centre of its base. Strings are attached to the cone at P and at O. The hangs in equilibrium with PO horizontal and the string taut. The strings attached at P and O make angles of 0° and 20° with the vertical.

(Sketch shows a cross-section)

(i) By taking moment about P for the cone, find the tension in the string attached at O.

(ii) Find the value of θ and the tension in the string attached at P.

Solution: Let G be the centre of mass of the cone.

(i) and h is its height.

Then, \( PG = \frac{3}{4} h \) and \( OG = \frac{h}{4} \)

Taking moment about P:

\[ W \times PG = T \times G320 \times h/4 \]

\[ 50 \times \frac{3}{4} h = T \times G320 \times \frac{h}{4} \]

\[ \Rightarrow \frac{T}{4} = \frac{3 \times 50}{4 \times G320} \]

\[ \Rightarrow T = 39.9 \text{ N} \]

(ii) As the cone is in equilibrium, the sum of horizontal components be zero or \( T \sin θ = T \times G320 = 39.9 \times G320 \)

\[ \Rightarrow T \sin θ = 13.64 \text{ N} \quad i \]

Now taking moment about O,

\[ T \times G320 \times h = 50 \times h/4 \]

\[ \Rightarrow T \times G320 = 12.5 \quad \text{(2)} \]

Divide (1) by (2)

\[ \frac{T \sin θ}{T} = \frac{13.64}{12.5} = 1.0917 \]

\[ \Rightarrow \sin θ = 1.0917 \]

\[ \Rightarrow θ = 47.5° \]

From (2) \( T \times G320 = 12.5 \)

\[ \Rightarrow T \times G320 = 12.5 \]

\[ \Rightarrow T = 18.5 \text{ N} \]
Example 25: A uniform solid cone has weight 5 N and base radius 0.1 m. AB is a diameter of the base of the cone. The cone is held in equilibrium with A in contact with a rough horizontal surface and AB vertical, by a force applied at B. The force has magnitude 3 N and acts parallel to the axis of the cone. Calculate the height of the cone.

Solution: Let the height of the cone = h

Let the centre of mass of cone at G, OG = \( \frac{h}{4} \) where O is the centre of similar base.

Taking moment about A,

\[
5 \times \frac{h}{4} = 3 \times 0.2
\]

\[
\Rightarrow h = 0.48
\]

height of cone = 0.48 m
Equilibrium of a rigid body

Sliding and Toppling

- The object is on the point of sliding, then
  the force of friction \( F = \mu R \).
  \( R \) is normal contact force between the two bodies and
  \( \mu \) is the coefficient of friction.
  or \( \mu = \frac{F}{R} \).

- Rough inclined plane:
  If a body is placed on a rough inclined
  plane in the point of sliding down
  the plane under the action of its
  weight and the reactions of the plane.
  Only, the angle of inclination of the
  plane to the horizontal \( \theta \) is equal
  to the angle of friction.

  Resolving parallel and perpendicular to the plane.
  Parallel to plane: \( W \sin \theta = \mu R \)
  Perpendicular to plane: \( W \cos \theta = R \)

  \( \Rightarrow \tan \theta = \mu \) coefficient of friction
  where \( \theta \) is called the angle of friction.

Toppling:

- About to topple about
  the pivot edge \( A \).
- Toppling about
  the pivot edge \( A \).
Example 36: ABC is the cross-section through the centre of mass of a uniform prism, which rests on a rough horizontal surface, AB = 0.4 m and C is 0.9 m above the surface.

The prism is on the point of toppling about its edge through B. A = 0.4 B

(i) Show that angle $\angle BAC = 48.4^\circ$ correct to 3 sf. 

A force of magnitude 18 N acting in the plane of cross-section and perpendicular to AC is now applied to the prism at C. The prism is on the point of rotating about its edge through A.

(ii) Calculate the weight of the prism.

(iii) Given also that the prism is on the point of slipping, calculate the coefficient of friction between the prism and the surface.

Solution:

Let $G$ be the COM of the prism, $M$ be the mid-point of $AB$.

(i) $G M = \frac{G B}{C M} = \frac{0.2}{0.19}$

$CE = 0.9$

Draw $CEAB$

$\Rightarrow GB = 0.3 m$, $MB = 0.2 m$

Since the prism is on the point of toppling about B, $AE = AM + ME = 0.8$

$G$ is the vertical through $COM$'s mass, pass through $B$, $AB = CE = 0.7$

$\Rightarrow AN = 0.8$

In $\triangle ABC$, $CE = 0.8 A$

From $(iii)$, horizontal component of force $F = 18.8 N = 18 \times 0.8 = 14.46$

In $\triangle A CE$, $CE = 0.8 A$

Now, taking moment about $A$

$18 \times AC = WXAB \Rightarrow 18 \times 1.2041 = WX \times 0.4$

$\Rightarrow W = 54.3 N$

$\Rightarrow \mu = \frac{F}{W} = 0.265$
Example 27: An object is formed by joining a hemispherical shell of radius 0.2 m and a solid cone, with base radius 0.2 m and height h m, along the circumference. The centre of mass, G, of the object is h m from the vertex of the cone on the axis of symmetry of the object. The object is in equilibrium on a horizontal plane, with the curved surface of the cone in contact with the plane, the object is on the point of toppling.

(i) Show that:
\[ d = \frac{h + 0.04}{h} \]  -- [3]

(ii) It is given that the cone is uniform and of weight 4 N, and the hemispherical shell is uniform and of weight 8 N. Given also that h = 0.8, find W. [W = 15.53/66] -- [6]

Solution: As the object is on the point of toppling the vertical through COM 'G' will pass through the edge N of the object...

Let MN be the centre of circular base,
G N is vertical, MN = 0.2 (radius)
Let OM = h, OG = d, \( \theta \leq \theta \angle GON = \theta \)
(i) In \( \triangle AGN \), ON = GS = GN
\[ \Rightarrow ON = d \times 30 \]  
and \( \text{cm} \), OMN,
\[ \frac{cm}{ON} = GS = \frac{h}{ON} = \frac{h}{GN} = \frac{h}{ON} = \frac{h}{ON} \]  -- [2]

\[ \text{from (1) + (2)} \]
\[ \frac{cm}{GS} = \frac{\theta}{GS} = \frac{d}{GS} \]
\[ \text{Now,} \quad ON = \sqrt{h^2 + 0.04} \]
\[ \Rightarrow \text{from (3) \frac{\theta}{GS} = \frac{\theta}{GS} = \frac{\theta}{GS} = \frac{\theta}{GS} \]  -- [4]

\[ \text{from (3) + (4)} \]
\[ d = \frac{h}{\sqrt{h^2 + 0.04}} \]
\[ = 0.04 + 0.8 \]
\[ \Rightarrow d = \frac{h + 0.04}{h} \]

(ii) COM of cone from O = \( \frac{3}{4} h = \frac{3 \times 0.8}{4} = 0.6 \)
COM of hemisph = \( \frac{h + 0.8}{2} = 0.4 + 0.8 = 0.8 \)

\[ \Rightarrow 0.6 + 0.4 + 0.9 \times W = d(4 + W) \]  -- [5]
\[ d = \frac{h + 0.04}{h} = \frac{0.85}{0.8} \]
\[ \Rightarrow 2.4 + 0.9 \times W = 0.85(4 + W) \]  -- [5]
Example 28: A cylindrical container is open at the top. The curved surface and the circular base of the container are both made from the same thin uniform material. The container has radius 0.2 m and height 0.9 m.

(i) Show that the centre of mass of the container is 0.405 m from the base.

The container is placed with its base on a rough inclined plane. The container is in equilibrium at the point of slipping down the plane and also on the point of toppling.

(ii) Find the coefficient of friction between the container and the plane.

Solution: \( M = 2\pi rh + \pi h^2 = \pi \times 0.2 \times 0.9 + \pi \times 0.2^2 \)

\( = (\pi \times 0.2 \times 0.9 + \pi \times 0.2^2) \times \overline{x} \)

\( = (\pi \times 0.2 \times 0.9 + \pi \times 0.2^2) \times 0 \) (distance from 0, the centre of circular base)

\( \Rightarrow \overline{x} = 0.405 \text{ m} \Rightarrow OG = 0.405 \text{ m} \)

\( G \) is the centre of mass.

When the object is at the point of slipping, then \( \mu = \tan \theta \).

and the object at the point of toppling \( G \) is vertical.

\( \mu = \frac{R - W \sin \theta}{W \cos \theta} \Rightarrow \mu = \tan \theta = \frac{OG}{OG} \)

\( = \frac{0.9}{0.405} \)

\( \mu = 0.494 \)
Equilibrium of a rigid body.
Toppling but not sliding.

Example 29: A uniform solid cone has a height of 30 cm and base radius 4 cm. The cone is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted and the cone remains in equilibrium until the angle of inclination of the plane reaches 35°, when the plane topples. The diagram shows a cross-section of the cone.

(i) Find the value of r.
(ii) Show that the coefficient of friction between the cone and the plane is greater than 0.7.

Solution: COM of cone is at a distance \( \frac{3h}{4} \) from O.

\[ GM = \frac{3h}{4} = 7.5 \text{ cm} \]

as the plane topples when \( \theta = 35° \)
at this moment \( GA \) is vertical.
angle \( AGA = \theta = 35° \)

\[ l \cos \theta = AM \Rightarrow \frac{h}{GM} = \tan 35° \]

\[ \Rightarrow h = 7.5 \times \tan 35° = 5.25 \text{ cm} \]

\[ \Rightarrow r = 5.25 \text{ cm} \]

(Note: when an object is at the point of sliding and toppling both \( \mu = \tan \theta \), \( GA \) in vertical)

In this case as the object topples but does not slide \( \mu > \tan \theta \Rightarrow \mu > \tan 35° \)

\[ \Rightarrow \mu > 0.71 \]
Equilibrium of a rigid body

Example 30: Fig (A) shows the cross-section (A) of a uniform solid. The cross-section has the shape and dimensions shown.

The centre of mass C of the solid lies in the plane of this cross-section.

The distance of C from DF is $y\text{cm}$.

(i) Find the value of $y$.

The solid is placed on a rough plane. The coefficient of friction between the solid and the plane is $\mu$.

The plane is tilted so that EF lies along a line of greatest slope.

(ii) The solid is placed so that $F$ is higher up the plane than $E$. When the angle of inclination is sufficiently great the solid starts to topple. Show that $\mu > \frac{1}{2}$.

(iii) The solid is now placed so that $F$ is higher up than $E$ when the angle of inclination is sufficiently great the solid starts to slide (without toppling). Show that $\mu < \frac{5}{6}$.

Solution: BL and OP are produced to meet DE at $M$ and $R$ respectively.

Now the rectangles $ABMD$, $LMRP$ and $QREL$ each is of size $20 \times 5 \text{cm}^2$, at $C$ to $y$, $y\text{cm}$ in the $y\text{cm}$.

All distances measured from $DE$.

\[ y(20 \times 5 + 20 \times 5 + 20 \times 5) = (20 \times 5) \times 10 + (20 \times 5 \times 20 \times 5 + (20 \times 5) \times 10 \]

\[ 300y = 2250 \]

\[ y = \frac{2250}{300} = 7.5 \text{ cm} \]

or $CG = 7.5 \text{ cm}$

(Continued...)
Given that when the plane is inclined at an angle $\theta$ to the horizontal, the object is at the point of toppling at edge $E$, $CE$ is vertical, $COM'$ is at $C$, and that $CG = 7.5$ cm.

Draw $CS \perp EF \Rightarrow ES = 7.5 \cdot \frac{30}{2} = 15$ cm.

$\angle ECS = 0$

In $\triangle CSE$

$$\tan \theta = \frac{ES}{CS} = \frac{7.5}{15} = \frac{1}{2}$$

Now as for this value of $\theta$, the object is at the point of toppling but not sliding $\Rightarrow \mu > \frac{1}{2}$

(iii) Now for toppling, $FS = 20 - 7.5 = 12.5$

$$\mu = \tan \theta = \frac{FS}{CS} = \frac{12.5}{15}$$

$$(CF \perp \text{Vertical}) \hspace{2cm} \tan \theta = \frac{5}{6}$$

For this value of $\tan \theta$, the object will topple.

But given that object slide without toppling $\Rightarrow \mu < \tan \theta$

$\Rightarrow \mu < \frac{5}{6}$