"P_1" **Binomial Theorem** $\boxed{\text{Notes}}$

Combination (or Selection):

Combination of n different objects taken r at a time denoted by: $r \leq n$

$$\binom{n}{r} \text{ or } {}^nC_r = \frac{n!}{r!(n-r)!}$$

[n is a non-negative Integer]

Note:

(i) ${}^nC_n = {}^nC_0 = 1$

(ii) ${}^nC_1 = n$

(iii) ${}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2\times1}$

For Example:

(i) ${}^7C_0 = {}^7C_7 = 1$

(ii) ${}^7C_1 = 7$

(iii) ${}^7C_2 = \frac{7!}{2!\,5!} = \frac{7\times6}{2\times1} = 21$

(iv) ${}^{25}C_3 = \frac{25!}{3!\,22!} = \frac{25\times24\times23}{3\times2\times1}$

Imp. Result: ${}^nC_r = {}^nC_{n-r}$ $\left[{}^7C_2 = {}^7C_5 \right.$ Example.

Also: ${}^7C_5 = \frac{7!}{5!\,2!} = \frac{7!}{2!\,5!} = {}^7C_2$

"P_1" **Binomial Theorem:** $\boxed{\text{Notes}}$

$(x+a)^1 = x + a$

$(x+a)^2 = x^2 + 2x \cdot a + a^2$

$(x+a)^3 = x^3 + 3x^2 a + 3x a^2 + a^3$

$(x+a)^4 = x^4 + 4x^3 a + 6x^2 a^2 + 4x a^3 + a^4$

$(x+a)^5 = x^5 + 5x^4 a + 10x^3 a^2 + 10x^2 a^3 + 5x a^4 + a^5$

$(x+a)^6 = x^6 + 6x^5 a + 15x^4 a^2 + 20x^3 a^3 + 15x^2 a^4 + 6x a^5 + a^6$

$= {}^6C_0 x^6 + {}^6C_1 \text{--} + {}^6C_2 \text{--} + {}^6C_3 \text{--} + {}^6C_4 \text{--} + {}^6C_5 \text{--} + {}^6C_6 a^6$

Binomial Theorem: for n is a non-negative integer

(i) $(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \cdots + {}^nC_r x^{n-r} a^r + \cdots + {}^nC_n a^n$

General Term $= {}^nC_r \cdot x^{n-r} a^r \; ; \; r \le n$

(ii) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_r x^r + \cdots + {}^nC_n \cdot x^n$

General Term $= {}^nC_r \cdot x^r$

Example 1 (i) Find the term independent of x in the expansion of:

$$\left(2x^2 - \frac{3}{x^3}\right)^{25}$$

In $(x+a)^n$ Gen. Term $= {}^nC_r\, x^{n-r} a^r$

General Term $= {}^{25}C_r (2x^2)^{25-r} \cdot \left(-\frac{3}{x^3}\right)^r$

$= {}^{25}C_r \cdot 2^{(25-r)} \cdot (-3)^r \cdot (x^2)^{25-r} \cdot (x^{-3})^r$

$= (-1)^r \cdot {}^{25}C_r \cdot 2^{25-r} \cdot 3^r \cdot x^{50-2r} \cdot x^{-3r}$

$= (-1)^r \times {}^{25}C_r \cdot 2^{25-r} \times 3^r \times x^{50-5r}$ —— (1)

Now for the term independent of x;
Exponent of x in (1) should by zero

$50 - 5r = 0 \Rightarrow r = 10$

from (1) Req. independent x term:

$= (-1)^{10} \cdot {}^{25}C_{10} \cdot 2^{15} \cdot 3^{10}$

$= {}^{25}C_{10} \cdot 2^{15} \cdot 3^{10}$

(ii) Also find the coefficient of x^{15} in $\left(2x^2 - \frac{3}{x^3}\right)^{25}$.

for coeff of x^{15}, put exponent of x in (1) 15,

$50 - 5r = 15 \Rightarrow r = 7$

$\therefore$ from (1) Coeff of $x^{15} = (-1)^7 \cdot {}^{25}C_7\, 2^{18} \cdot 3^7$

$= -{}^{25}C_7\, 2^{18} \cdot 3^7$

Example 2: $\boxed{SP\text{-}2017/01/Q1}$

In the expansion of $\left(1-\frac{2x}{a}\right)(a+x)^5$, where a is a non-zero constant, show that the coeff of x^2 is zero.

Solution: $(a+x)^5 = a^5 + 5a^4x + 10a^3x^2 + \text{---}$ (i)

Now in $\left(1-\frac{2x}{a}\right)(a+x)^5$

$$= \left(1-\frac{2x}{a}\right)\left[a^5+5a^4x+10a^3x^2+--\right] \text{ fr (i)}$$

Terms involving $x^2 = 1\times 10a^3x^2 - \frac{2x}{a}\times 5a^4x$

$$= 10a^3x^2 - 10a^3x^2$$

$$= (10a^3 - 10a^3)x^2$$

$$= 0\cdot x^2$$

$\therefore$ Coeff of $x^2 = 0$.

Example 3 $\boxed{M\text{-}17/12/Q2}$

In the expansion of $\left(\frac{1}{ax}+2ax^2\right)^5$ the coeff. of x is 5. Find the value of a.

Solution:

$\left(\frac{1}{ax}+2ax^2\right)^5$ Gen. Term $= {}^5C_r\left(\frac{1}{ax}\right)^{5-r}\cdot(2ax^2)^r$

$$= {}^5C_r\cdot\left(\frac{1}{a}\right)^{5-r}\cdot(2a)^r\cdot x^{r-5}\cdot x^{2r}$$

$$= {}^5C_r\cdot a^{r-5}\cdot 2^r a^r\cdot x^{3r-5}$$

$\therefore$ Coeff of $x = {}^5C_2\cdot 2^2\cdot a^{2r-5}$ $\left[\text{for Coeff of } x^1 \Rightarrow 3r-5=1,\ r=2\right.$

$= 10\times 4\times a^{-1} = 5$ (given)

$\Rightarrow a = 8$ ✓