"P1"

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Pure Maths-1
9709

Quadratics
Exercise (June 2017 to March 2019)
(With marking scheme)

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"P1"

Quadratics

1.(i) Express x^2-4x+7 in the form $(x+a)^2+b$ ---[2]

The function f is defined,

$f(x) = x^2-4x+7$ for $x < k$, where k is a constant.

(ii) State the largest value of k for which f is a decreasing function. ---[1]

The value of k is now given to be 1.

(iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. ---[3]

(iv) The function g is defined by $g(x) = \frac{2}{x-1}$ for $x>1$, Find an expression for $gf(x)$ and state the range of gf. [M-19/12/Q8] --[4]

2. The equation of the curve is $y = x^2-6x+k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. ---[2]

(ii) Find the value of k for which the line $y=2x+7$ is tangent to the curve. [S-18/12/Q2] --[3]

3. Express $3x^2-12x+7$ in the form $a(x+b)^2+c$, where a, b and c are constants. [S-18/13/Q1] --[3]

4. Showing all necessary working, solve the equation,

$$4x - 11x^{1/2} + 6 = 0$$

[W-18/11/Q1] ---[3]

5. A line has equation $y = x+1$ and a curve has equation $y = x^2+bx+5$, Find the set of values of the constant b for which the line meets the curve. [W-18/11/Q2] ---[4]

6. A curve has equation $y=2x^2-3x+1$ and a line has equation $y = kx+k^2$, where k is a constant.

(i) Show that for all values of k, the curve and line meet. ---[4]

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve. ..[4]

[W-18/13/Q9]

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7. Find the set of values of a for which the curve $y=-\frac{2}{x}$ and the straight line $y=ax+3a$ meet at two distinct points. [W-17/13/Q2] --[4]

Answers

1\. (i) $(x-2)^2+3$

(ii) Largest k is 2 $(k\leq 2)$

(iii) $f(x)=(x-2)^2+3$

$\Rightarrow\quad x-2=\pm\sqrt{y-3}$

$\Rightarrow f^{-1}(x)=2-\sqrt{x-3}$ for $x>4$

(iv) $gf(x)=\frac{2}{x^2-4x+7-1}=\frac{2}{(x-2)^2+2}$

since $f(x)>4 \Rightarrow gf(x)<\frac{2}{3}$ (or since $x<1$)

Range of $gf(x)$ is $0<gf(x)<2/3$

2\. (i) $(x-3)^2+k-9>0$, $k-9>0$

or $2x-6=0\rightarrow(3,k-9)$, $k-9>0$

or $b^2-4ac<0\rightarrow 36<4k$

$\rightarrow k>9$ ✓

(ii) $x^2-6x+k=7-2x$

$\rightarrow x^2-4x+k-7=0$

$\rightarrow 16-4(k-7)=0$ [$b^2-4ac=0$

$\rightarrow k=11$

3\. $3(x-2)^2-5$

4\. $(4x^{1/2}-3)(x^{1/2}-2)=0$

$x^{1/2}=3/4$ or 2

$x=9/16$ or 4

5\. $x^2+bx+5=x+1$

$\rightarrow x^2+(b-1)x+4=0$

$(b-1)^2-16\geq 0$ [$b^2-4ac\geq 0$]

$\rightarrow b\leq\ .\ , b\geq 5$

6 (i) $2x^2-3x+1=kx+k^2$ (1)

$2x^2-(3+k)x+(1-k^2)=0$

$b^2-4ac=(3+k)^2-4\times 2(1-k^2)$

$\rightarrow =9k^2+6k+1$

$=(3k+1)^2\geq 0$ for all k.

∴ Intersect. ✓

(ii) for tangent

$(3k+1)^2=0$ ($b^2-4ac=0$

$\rightarrow k=-\frac{1}{3}$ ✓

for $k=-\frac{1}{3}$ in (1) $2x^2-8/3\,x+8/9=0$

$\rightarrow x^2-4/3\,x+4/9=0$

$(x-2/3)^2=0\rightarrow x=\frac{2}{9}, y=-\frac{1}{9}$

$(\frac{2}{3},-\frac{1}{9})$ ✓

7\. $ax+3a=-\frac{2}{x}$

$ax^2+3ax+2=0$

[$b^2-4ac>0$

$B^2-4ac=9a^2-4a\times 2>0$

$a(9a-8)>0$

$\rightarrow a<0$ or $a>8/9$ ✓

$\longleftarrow$ X — X $\longrightarrow$