(P3) Differentiating and Integrating $\boxed{\text{Notes}}$ Page - 1

(i) Exponential and Logarithms functions:

1.(i) $y = e^x$

$\frac{dy}{dx} = e^x$

$\boxed{\text{where } e = 2.718 \text{ (approx) and } \lim_{h\to 0} \frac{e^h - 1}{h} = 1}$

(ii) $\frac{d}{dx} e^{ax+b} = e^{ax+b} \times a$

(iii) $\frac{d}{dx} e^{x^2} = e^{x^2} \cdot 2x$ (Chain Rule)

(iv) $\frac{d}{dx} b^x = b^x \times \ln b.$

$(\because b^x = e^{x \ln b})$

2.(i) $\int e^x \, dx = e^x + k$

(ii) $\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + k$

The Natural Logarithm

1.(i) $y = e^x \Leftrightarrow \log_e y = x \quad ; x \in R, y \in R, y > 0$

or $\boxed{\ln x = \log_e x \quad \text{where } x \in R, x > 0}$

2.(i) $\frac{d}{dx} \ln x = \frac{1}{x}$

(ii) $\frac{d}{dx} \ln(ax+b) = \frac{a}{(ax+b)}.$

3.(i) $\int \frac{1}{x} dx = \ln x + k \quad ; x > 0$

(ii) $\int \frac{1}{(ax+b)} dx = \frac{1}{a} \ln(ax+b) + k$

where $(ax+b) > 0$

Imp

3(iii) $\int \frac{1}{x} dx = \ln(-x) + k$

when $x < 0$

(iv) $\int \frac{1}{(ax+b)} dx = \frac{1}{a} \ln(-ax-b) + k$

when $(ax+b) < 0$

Example

$\int_0^1 \frac{1}{(x-2)} dx = \left[\ln(2-x)\right]_0^1 = \underline{-\ln 2}$

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(ii) Trigonometric Functions.

1. $\frac{d}{dx} \sin x = \cos x$
2. $\frac{d}{dx} \cos x = -\sin x$
3. $\frac{d}{dx} \tan x = \sec^2 x$
4. $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$
5. $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$
6. $\frac{d}{dx} \sec x = \sec x \cdot \tan x$

7 (i) $\frac{d}{dx} \sin(ax+b) = a\cos(ax+b)$

(ii) $\frac{d}{dx} \ln \sin x = \cot x$

(iii) $\frac{d}{dx} \ln \cos x = -\tan x$

(iv) $\frac{d}{dx} \sin^4 x = 4\sin^3 x \cdot \cos x$ $\leftrightarrow$

diff → using Chain Rule.

$\left(\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$

1'. $\int \cos x \, dx = \sin x + k$

2'. $\int \sin x \, dx = -\cos x + k$

3'. $\int \sec^2 x \, dx = \tan x + k$

4'. $\int \operatorname{cosec}^2 x \, dx = -\cot x + k$

5'. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + k$

6'. $\int \sec x \tan x \, dx = \sec x + k.$

7' (i) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + k$

(ii) $\int \cot x \, dx = \ln \sin x + k$

(iii) $\int \tan x \, dx = -\ln \cos x + k$ or $\ln \sec x + k$

(iv) $\int \sin^3 x \cdot \cos x \, dx$ — Integration using Substitution "

Put $u = \sin x$, $du = \cos x \, dx$

$= \int u^3 \, du$

$= \frac{u^4}{4} = \frac{1}{4}\sin^4 x + k$

Trigonometry for Integration:

1 (i) $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

(ii) $\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x)$

2. (i) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

(ii) $\cos^2 \frac{x}{2} = \frac{1}{2}(1 + \cos x)$

3. $\tan^2 x = \sec^2 x - 1$

4. $\cot^2 x = \operatorname{cosec}^2 x - 1$

5. $\boxed{\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]}$

6. $\sin^3 x = \frac{1}{4}[3\sin x - \sin 3x]$
   (as $\sin 3x = 3\sin x - 4\sin^3 x$)

7. $\cos^3 x = \frac{1}{4}[3\cos x + \cos 3x]$
   (as $\cos 3x = 4\cos^3 x - 3\cos x$)

(iii) Product Rule and Quotient Rule of differentiation:

1, Sum Rule: if $y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

2. Product Rule:

$$y = u \cdot v$$

$$\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$$

3. Quotient Rule:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$$

4. $y = u \cdot v \cdot w$

$$\frac{dy}{dx} = vw\frac{du}{dx} + uw\frac{dv}{dx} + uv\frac{dw}{dx}$$

5. $y = \frac{1}{f(x)}$

$$\frac{dy}{dx} = \frac{-f'(x)}{[f(x)]^2}$$

Examples: Find the derivative of the following functions.

Q1. $y = x^3 \cdot \sin x$

$$\frac{dy}{dx} = 3x^2 \sin x + x^3 \cos x$$

Q2. $y = \frac{\cos x}{\sqrt{x}}$

$$\frac{dy}{dx} = \frac{\sqrt{x} \cdot \frac{d}{dx}\cos x - \cos x \cdot \frac{d}{dx}\sqrt{x}}{(\sqrt{x})^2}$$

$$= \frac{-\sqrt{x}\sin x - \cos x \cdot \frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{-(2x\sin x + \cos x)}{2x\sqrt{x}}$$

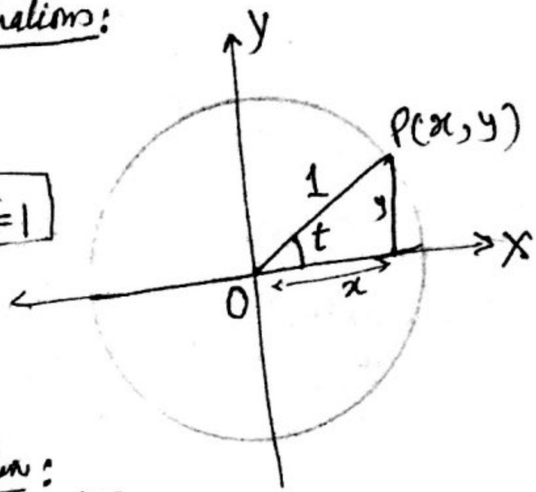
Q3. $y = \sin^5 x \cdot \cos^3 x$

$$\frac{dy}{dx} = \sin^5 x \cdot \frac{d}{dx}\cos^3 x + \cos^3 x \cdot \frac{d}{dx}\sin^5 x$$

$$= \sin^5 x \cdot 3\cos^2 x\,(-\sin x) + \cos^3 x \cdot 5\sin^4 x \cdot \cos x$$

$$= \sin^4 x \cdot \cos^2 x\,(5\cos^2 x - 3\sin^2 x)$$

(iv) differentiation of Parametric Equations:



1. Parametric Equations

Parametric equation of a circle with centre at O(0,0) and radius 1. is $\boxed{x^2+y^2=1}$

$$\begin{cases} x = \cos t & \text{and} \\ y = \sin t \end{cases}$$

2. differentiation of Parametric equation:

$y = f(t)$ and $x = g(t)$

$\frac{dy}{dt} = ✓$; $\frac{dx}{dt} = ✓$

$$\boxed{\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}}$$

Example 1. Find $\frac{dy}{dx}$;

given $x = t^3$ and $y = 2t$

$\frac{dx}{dt} = 3t^2$ & $\frac{dy}{dt} = 2$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2}{3t^2} ✓$$

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(v) Differentiation of Implicit Equation:

Example (i) Find $\frac{dy}{dx}$;

for $3x^2 - 2y^3 = 1$

diff. w.r.t x

$$6x - 6y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y^2} \checkmark$$

(ii) Find the equation of tangent to the curve $x^2 - 2xy + 2y^2 = 5$ at the point $(1,2)$

Given $x^2 - 2xy + 2y^2 = 5$

diff. w.r.t x

$$2x - 2\left(x\frac{dy}{dx} + y\right) + 4y\frac{dy}{dx} = 0$$

$$2\frac{dy}{dx}(2y - x) = 2(y - x)$$

$$\frac{dy}{dx} = \frac{y-x}{2y-x}$$

$$\left(\frac{dy}{dx}\right)_{(1,2)} = \frac{2-1}{2\times2-1} = \frac{1}{3} \checkmark$$

$\therefore$ Eqn of tangent at $(1,2)$

$$y - 2 = \frac{1}{3}(x-1)$$

or $\underline{3y = x + 5}$ $\checkmark$

Note $u = f(y)$

$$\frac{du}{dx} = f'(y)\cdot\frac{dy}{dx}$$

(i) Example

$$\frac{d}{dx}y^2 = 2y\frac{dy}{dx}$$

(ii) $\frac{d}{dx}\sin y = \cos y\frac{dy}{dx}$

(iii) $\frac{d(xy)}{dx} = \left(x\frac{dy}{dx} + y\right)$