P3

Pure Maths. 3.

Polynomials.

Exerise - 1.

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Polynomials

Q1. The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b. --[5]

[S-18/31/Q4]

Q2. Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$ --[3]

[W-17/31/Q1]

Q3. The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$.

(i) Find the value of a --[2]

(ii) When a has this value,

(a) factorise $p(x)$ --[2]

(b) Solve the inequality $p(x) > 0$, justifying your answer. --[3]

[M-16/32/Q4]

Q4. The polynomial $4x^4 + ax^2 + 11x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 2$

(i) Find the values of a and b --[5]

(ii) When a and b have these values, find the real roots of the equation $p(x) = 0$ [W-16/33/Q4] --[2]

Q5. The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x+1)$ the remainder is 1.

(i) Find the values of a and b --[5]

(ii) When a and b have these values, factorise $p(x)$ completely. --[3]

[W-15/31/Q6]

Q6. Using factor theorem, factorise completely, --[4

$16x^3 - 24x^2 - 15x - 2$ [S-14/31/Q6(ii)]

(*) Q7. (i) The polynomial $f(x)$ is of the form $(x-2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x-2)$ is a factor of $f'(x)$. --[2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x-2)^2$. Using factor theorem and the result of part (i) or otherwise, find the values of a and b. [S-14/32/Q5] --[5]

(P3)

Polynomials

Answers

Q1. $a=1$ and $b=2$

Q2. quotient $= x^2-2x+5$

remainder $= -12x+5$

Q3 (i) $a=3$

(ii)(a) $(2x+1)(2x^2-x+2)$

(b) critical value $x=-\frac{1}{2}$

as $(2x^2-x+2)=2\left[(x-\frac{1}{4})^2+\frac{15}{16}\right]$

is always positive.

$\therefore p(x)>0 \Rightarrow x>-\frac{1}{2}$ ✓

Q4 (i) $a=1$ and $b=-6$

(ii) $p(x)=(x^2-x+2)(4x^2+4x-3)$

$x^2-x+2=0$ has no real roots.

for $4x^2+4x-3=0$

real roots are $\frac{1}{2}$ and $-\frac{3}{2}$ ✓

Q5 (i) $a=6$ and $b=-3$

(ii) $(x+1)(8x^2-2x-1)$

or $(x+1)(4x+1)(2x-1)$ ✓

Q6. $(x-2)(4x+1)^2$

Q7. (i) $f'(x)=(x-2)^2g'(x)+2(x-2)g(x)$

$= (x-2)\left[(x-2)g'(x)+2(x-2)g(x)\right]$

$\therefore (x-2)$ is a factor of $f'(x)$. ✓

(ii) $P(2)=0 \Rightarrow 32+16a+24+4b+a=0$

$\Rightarrow 17a+4b=-56$ ①

(continued →)

Q7(ii) $P'(x)=5x^4+4ax^3+9x^2+2bx$

from Part (i)

$(x-2)$ is also a factor of $P'(x)$

$\Rightarrow P'(2)=0$

$\Rightarrow 80+32a+36+4b=0$

$\Rightarrow 32a+4b=-116$

or $8a+b=-29$ ②

Solving ① & ②

$a=-4$ & $b=3$

← X — X →