[P3]   *EXERCISE 1*

A – LEVEL: POLYNOMIALS, MODULUS, EXPONENTS, LOGARITHMS

Q1. Solve the inequality $|2x-5| > 3|2x+1|$.   [4]

Q2. Using the substitution u=3^x, solve the equation, $3^x + 3^{2x} = 3^{3x}$, giving your answer correct to 3 significant figures.   [5]

Q3.The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by p(x). It is given that (x+1) is a factor of p(x) and that when p(x) is divided by (2x+1), the remainder is 1.

   (i)   Find the value of a and b.   [5]
   (ii)   When a and b has these values, factorise p(x) completely.   [3]

                                                         [SP-2017/O3/Q1,Q2,Q6
                                                              W-15/31/32]

Q4. Solve the equation $\ln(x^2 + 4) = 2lnx + ln4$, giving your answer in an exact form.

Q5. The polynomial $4x^3 + ax + 2$, when x is a constant, is denoted by p(x). It is given that (2x+1) is a factor of p(x).

   (i) Find the value of a.   [2]
   (ii)When a has this value,
       (a) Factorise p(x)   [2]
       (b) Solve the inequality p(x)>0, justify your answer.   [3]

                                                         [M-16/32/Q1,Q4]

Q6.

(i) Solve the equation $2|x-1|=3|x|$ [3]

(ii) Hence solve the equation $2|5^x-1|=3|5^x|$, giving your answer correct to 3 significant figures. [2]

[S-16/31/Q1]

Q7. Using logarithms solve the equation $4^{3x-1}=35^x$, giving your answer correct to 3 decimal places. [4]

[S-16/32/Q1]

Q8. Solve the inequality $2|x-2|>|3x+1|$ [4]

Q9. The variable x and y satisfy the relation, $3^y=4^{2-x}$

(i) By taking logarithms, show that the graph of y against x is straight line, state the exact value of the gradient of this line. [3]

(ii) Calculate the exact x-coordinate of the point of intersection of this line with the line with equation y=2x, simply your answer. [2]

[S-16/33/Q2,Q1]

Q10. Solve the equation: $\dfrac{3^x+2}{3^x-2}=8$, giving your answer correct to 3 decimal places. [3]

[W-16/31/Q1

/32/]

Q11. It is given that z=ln(y+2)-ln(y+1). Express y in terms of z. [3]

Q12. The polynomial $4x^4+ax^2+11x+b$, where a and b are constants, is denoted by p(x). It is given that p(x) is divisible by x^2-x+2.

(i) Find the value of a and b. [5]

(ii) When a and b have these values, find the real roots of the equation p(x)=0. [2]

[W-16/33/Q1,Q4]

Q13. Sketch the graph of $y=e^{ax}-1$, where a is a positive constant. [2]

[W-15/33/Q1]

Q14. Use logarithms to solve the equation $2^{5x}=3^{2x+1}$, giving your answer correct up to 3 significant figures. [4]

[S-15/31/Q1]

Q15. Using substitution $u=4^x$, solve equation: $4^x+4^2=4^{x+2}$, giving your answer correct up to 3 significant figures. [4]

[S-15/31/Q2]

Q16. Solve the equation ln(x+4) = 2lnx + ln 4, giving your answer correct to 3 significant figures. [4]

Q17. Solve the inequality $|x-2| > 2x-3$ [4]

[S-15/33/Q1,Q2]

Q18. It is given that 2ln (4x-5)+ln (x+1)= 3ln3

(i) Show that $16x^3-24x^2-15x-2=0$ [3]

(ii) By first using factor theorem, factorise: $16x^3-24x^2-15x-2=0$ completely. [4]

(iii) Hence solve the equation 2ln(4x-5)+ln(x+1)=3ln3 [1]

[S-14/31/Q6]

Q19. Find the set of values of x satisfying the inequality:

$$|x+2a|>3|x-a|$$

Where a is a positive constant. [4]

Q20. Solve the equation: $2\ln(5-e^{-2X}) = 1$ giving your answer correct to 3 significant figures. [4]

[S-14/32/Q1,Q2]

Q21. Solve the equation $\log_{10}(x+9) = 2+\log_{10}x$ [3]

[S-14/33/Q1]

Q22. Use logarithms to solve the equation, $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places.

Q23. The polynomial ax^3+bx^2+x+3, where a and b are constants, is denoted by p(x). It is given that (3x+1) is a factor of p(x), and that when p(x) is divided by (x-2) the remainder of 21. Find the value of a and b.[5]

[W-14/31/Q1,Q3]

/32/]

Q24. Solve the inequality, $|3x-1|<|2x+5|$ [4]

Q25. The polynomial $4x^3+ax^2+bx-2$, where a and b are constants, is denoted by p(x). It is given that (x+1) and (x+2) are factors of p(x).

(i) Find the value if a and b. [4]
(ii) When a and b have these values, find the remainder when p(x) is divided by (x^2+1). [3]

[W-14/33/Q1,Q3]

Q26. Solve the equation, $2|3^x-1|=3^x$, giving your answer correct up to 3 significant figures. [4]

Q27. Given that $2\ln(x+4)-\ln x = \ln(x+a)$, express x in terms of a. [4]

Q28. The polynomial f(x) is defined by $f(x)=x^3+ax^2-ax+14$, where a is a constant. It is given that (x+2) is a factor of f(x).

   (i) Find the value of a. [2]
   (ii) Show that, when a has this value, the equation f(x) = 0 has only
      one real root. [3]

Q29. Find the quotient and reminder when $2x^2$ is divided by x+2. [3]

Q30. (i) Solve the equation $|4x-1|=|x-3|$ [3]

   (ii) Hence solve the equation $|4^{y+1}-1|=|4^y-3|$ correct to 3 significant figures. [3]

Q31. Solve the equation $|x-2|=\left|\frac{1}{3}x\right|$ [3]

Q32. The polynomial ax^3-20x^2+x+3, where a is a constant, is denoted by p(x). It is given that (3x+1) is a factor of p(x).

   (i) Find the value of a. [3]
   (ii) When a has this value, factorise p(x) completely. [3]

Q33. Solve the inequality $|4x+3|>|x|$ [4]

Q34. It is given that $\ln(y+1)-\ln y=1+3\ln x$. Express y in terms of x, in a form not involving logarithms. [4]

Q35. The polynomial $8x^3+ax^2+bx+3$, where a and b are constants, is denoted by p(x).It is given that (2x+1) is a factor of p(x) and that when p(x) is divided by (2x-1) the remainder is 1.

(i) Find the value of a and b. [5]
(ii) When a and b have these values, find the remainder when p(x) is divided by $2x^2-1$. [3]

[S-13/33/Q1,Q2,Q5]

Q36. Solve the equation $|4-2^x|=10$, giving your answer correct to 3 significant figures. [3]

Q37. The polynomial p(x) is divided by $p(x)=x^3-3ax+4a$, where a is constant.

(i) Given that (x-2) is a factor of p(x), find the value of a [2]
(ii) When a has this value,
  a) Factorise p(x) completely. [3]
  b) Find all the roots of the equation $p(x^2)=0$ [2]

[S-12/31/Q1,Q3]

Q38. Solve the equation $\ln(3x+4)=2\ln(x+1)$, giving your answer correct upto 3 significant figures. [4]

[S-12/32/Q1]

Q39. Solve the equation $\ln(2x+3)=2\ln x+\ln 3$, giving your answer correct to 3 significant figures. [4]

[S-12/33/Q2]

Q40. Solve $3|x-1|<|2x+1|$ [4]

Q41. Solve the equation, $5^{x-1} = 5^x - 5$, giving your answer correct to 3 significant figures. [4]
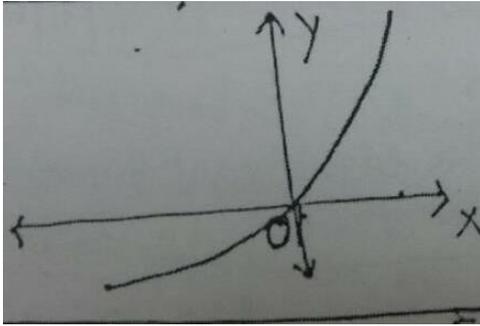
[W-12/33/Q1

/33/]

Q42. Solve the equation: $\ln(x+5) = 1+\ln x$, giving your answer in terms of e. [3]

[W-12/33/Q1]

ANSWERS

Q1. $-2 < x < \frac{1}{4}$

Q2. x = 0.438 (u=1.61)

Q3. (i) a=6 ; b= -3
    (ii)(x+1)(4x+1)(2x-1)

Q4. $x = 2/\sqrt{3}$

Q5. (i) a=3
    (ii) (a) $(2x+1)(2x^2 - x + 2)$
        (b) x > -1/2

Q6. (i) x = -2 and x = 2/5
    (ii) x = -0.569

Q7. x = 0.975

Q8. -5 < x < 3/5

Q9. (i) $-ln4/ln3$
    (ii) $x = ln4/ln6$

Q10. x = 0.860 $(3^x = 18/7)$

Q11. $y = \frac{2-e^z}{e^z - 1}$ $(e^z = \frac{y+2}{y+1})$

Q12. (i) a = 1 , b = -6
     (ii) ½ ; -3/2

Q20. x = -0.605 $[e^{-2x} = \frac{1}{(5-e^{\frac{1}{2}})}]$

Q21. x = 1/11

Q22. x = 22.28

Q23. a = 12 ; b = -20

Q24. -4/5 < x < 6

Q25. (i) a = 11 ; b = 5
     (ii)(x-13)

Q26. 0.631 and -0.369

Q27. $x = \frac{16}{a-8}$

Q28. (i) a = -1
     (ii) f(x) = $(x + 2)(x^2 - 3x + 7)$
     x= -2 is Only one real root   ;
        [as $b^2 - 4ac = -19$ < 0
          No real roots]

Q29. Quotient = $(2x - 4)$ and R= 8

Q30. (i) -2/3 or 4/5
     (ii) y = -0.161

Q31. x = 3/2

Q32. (i) a = 12
     (ii) (3x+1)(2x-1)(2x-3)

Q13.

Q14. x = 0.866

Q15. x = 0.0466 (u = 16/15)

Q16. x = 1.13 $[\frac{x+4}{x^2} = 4]$

Q17. x < 5/3

Q18. (ii) $(x - 2)(4x + 1)^2$
     (iii)x = 2   ;

              [x= - 1/4 not possible
    as $\ln(4x - 5)$ will not be defined]

Q19. $\frac{1}{4} a < x < \frac{5}{2} a$

Q33. x < -1 or x > -3/5
Q34. $y=(ex^3 - 1)^{-1}$

Q35. (i) a = -10 ; b = -1
     (ii) R = (3x-2)

Q36. 3.81

Q37. (i) a = 4
     (ii)$(x - 2)^2(x + 4)$
     (iii)$\pm\sqrt{2}$ and $\pm 2i$

Q38. x = 2.30 $(x^2 - x - 3 = 0)$

Q39. x = 1.39 $(3x^2 - 2x - 3 = 0)$

Q40. 2/5 < x < 4

Q41. x = 1.14 $(5^x = \frac{25}{4})$

Q42. x = $\frac{5}{(e-1)}$