EXERCISE 1

A – LEVEL: POLYNOMIALS, MODULUS, EXPONENTS, LOGARITHMS

Q1. Solve the inequality $|2x-5| > 3|2x+1|$.

Q2. Using the substitution $u=3^x$, solve the equation, $3^x + 3^{2x} = 3^{3x}$, giving your answer correct to 3 significant figures.

Q3. The polynomial $8x^3 + ax^2 + bx - 1$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $(x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x+1)$, the remainder is 1.

(i) Find the value of $a$ and $b$.

(ii) When $a$ and $b$ has these values, factorise $p(x)$ completely.

Q4. Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form.

Q5. The polynomial $4x^3 + ax + 2$, when $x$ is a constant, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$.

(i) Find the value of $a$.

(ii) When $a$ has this value,

(a) Factorise $p(x)$

(b) Solve the inequality $p(x) > 0$, justify your answer.
Q6.  
(i) Solve the equation $2|x-1|=3|x|$  \[3\]  
(ii) Hence solve the equation $2|5^x-1|=3|5^x|$, giving your answer correct to 3 significant figures.  \[2\]  
\[S-16/31/Q1\]  
Q7. Using logarithms solve the equation $4^{3x-1}=35^x$, giving your answer correct to 3 decimal places.  \[4\]  
\[S-16/32/Q1\]  
Q8. Solve the inequality $2|x-2|>|3x+1|$  \[4\]  
Q9. The variable $x$ and $y$ satisfy the relation, $3^y=4^{2-x}$  
(i) By taking logarithms, show that the graph of $y$ against $x$ is straight line, state the exact value of the gradient of this line.  \[3\]  
(ii) Calculate the exact $x$-coordinate of the point of intersection of this line with the line with equation $y=2x$, simply your answer.  \[2\]  
\[S-16/33/Q2,Q1\]  
Q10. Solve the equation: \[
\frac{3^x + 2}{3^x - 2} = 8
\]
giving your answer correct to 3 decimal places.  \[3\]  
\[W-16/31/Q1\]  
\[/32/\]  
Q11. It is given that $z=\ln(y+2)-\ln(y+1)$. Express $y$ in terms of $z$.  \[3\]
Q12. The polynomial $4x^4+ax^2+11x+b$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2-x+2$.

(i) Find the value of $a$ and $b$. [5]
(ii) When $a$ and $b$ have these values, find the real roots of the equation $p(x)=0$. [2]

W-16/33/Q1,Q4

Q13. Sketch the graph of $y=e^{ax}-1$, where $a$ is a positive constant. [2]

W-15/33/Q1

Q14. Use logarithms to solve the equation $2^{5x}=3^{2x+1}$, giving your answer correct up to 3 significant figures. [4]

S-15/31/Q1

Q15. Using substitution $u=4^x$, solve equation: $4^x+4^2=4^{x+2}$, giving your answer correct up to 3 significant figures. [4]

S-15/31/Q2

Q16. Solve the equation $\ln(x+4) = 2\ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]

Q17. Solve the inequality $|x-2| > 2x-3$ [4]

S-15/33/Q1,Q2

Q18. It is given that $2\ln (4x-5)+\ln (x+1)= 3\ln 3$

(i) Show that $16x^3-24x^2-15x-2=0$ [3]
(ii) By first using factor theorem, factorise: $16x^3-24x^2-15x-2=0$ completely. [4]
(iii) Hence solve the equation $2\ln(4x-5)+\ln(x+1)=3\ln 3$ [1]

S-14/31/Q6
Q19. Find the set of values of $x$ satisfying the inequality:

$$|x+2a| > 3|x-a|$$

Where $a$ is a positive constant. [4]

Q20. Solve the equation: $2\ln(5-e^{-2x}) = 1$ giving your answer correct to 3 significant figures. [4]

[S-14/32/Q1,Q2]

Q21. Solve the equation $\log_{10}(x+9) = 2 + \log_{10}x$ [3]

[S-14/33/Q1]

Q22. Use logarithms to solve the equation, $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places.

Q23. The polynomial $ax^3 + bx^2 + x + 3$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $(3x+1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x-2)$ the remainder of 21. Find the value of $a$ and $b$.[5]

[W-14/31/Q1,Q3]

Q24. Solve the inequality, $|3x-1| < |2x+5|$ [4]

Q25. The polynomial $4x^3 + ax^2 + bx - 2$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $(x+1)$ and $(x+2)$ are factors of $p(x)$.

(i) Find the value if $a$ and $b$. [4]

(ii) When $a$ and $b$ have these values, find the remainder when $p(x)$ is divided by $(x^2+1)$. [3]

[W-14/33/Q1,Q3]
Q26. Solve the equation, $2 |3^x-1|=3^x$, giving your answer correct up to 3 significant figures. [4]

W-13/31/Q2

Q27. Given that $2\ln(x+4)-\ln x = \ln(x+a)$, express $x$ in terms of $a$. [4]

Q28. The polynomial $f(x)$ is defined by $f(x)=x^3+ax^2-ax+14$, where $a$ is a constant. It is given that $(x+2)$ is a factor of $f(x)$.

(i) Find the value of $a$. [2]

(ii) Show that, when $a$ has this value, the equation $f(x) = 0$ has only one real root. [3]

W-13/33/Q1,Q3

Q29. Find the quotient and remainder when $2x^2$ is divided by $x+2$. [3]

Q30. (i) Solve the equation $|4x-1|=|x-3|$ [3]

(ii) Hence solve the equation $|4^{y+1}-1|=|4^y-3|$ correct to 3 significant figures. [3]

S-13/31/Q1,Q4

Q31. Solve the equation $|x-2|=\frac{1}{3}|x|$ [3]

Q32. The polynomial $ax^3-20x^2+x+3$, where $a$ is a constant, is denoted by $p(x)$. It is given that $(3x+1)$ is a factor of $p(x)$.

(i) Find the value of $a$. [3]

(ii) When $a$ has this value, factorise $p(x)$ completely. [3]

S-13/32/Q1,Q4

Q33. Solve the inequality $|4x+3|>|x|$ [4]
Q34. It is given that \( \ln(y+1) - \ln y = 1 + 3 \ln x \). Express \( y \) in terms of \( x \), in a form not involving logarithms. \( [4] \)

Q35. The polynomial \( 8x^3 + ax^2 + bx + 3 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (2x+1) \) is a factor of \( p(x) \) and that when \( p(x) \) is divided by \( (2x-1) \) the remainder is 1.

(i) Find the value of \( a \) and \( b \). \( [5] \)

(ii) When \( a \) and \( b \) have these values, find the remainder when \( p(x) \) is divided by \( 2x^2 - 1 \). \( [3] \)

[S-13/33/Q1,Q2,Q5]

Q36. Solve the equation \( |4-2^x| = 10 \), giving your answer correct to 3 significant figures. \( [3] \)

Q37. The polynomial \( p(x) \) is divided by \( p(x) = x^3 - 3ax + 4a \), where \( a \) is constant.

(i) Given that \( (x-2) \) is a factor of \( p(x) \), find the value of \( a \). \( [2] \)

(ii) When \( a \) has this value,
   a) Factorise \( p(x) \) completely. \( [3] \)
   b) Find all the roots of the equation \( p(x^2) = 0 \). \( [2] \)

[S-12/31/Q1,Q3]

Q38. Solve the equation \( \ln(3x+4) = 2\ln(x+1) \), giving your answer correct up to 3 significant figures. \( [4] \)

[S-12/32/Q1]

Q39. Solve the equation \( \ln(2x+3) = 2\ln x + \ln 3 \), giving your answer correct to 3 significant figures. \( [4] \)

[S-12/33/Q2]

Q40. Solve \( 3|x-1| < |2x+1| \). \( [4] \)
Q41. Solve the equation, \(5^{x-1} = 5^x - 5\), giving your answer correct to 3 significant figures. \[4\]

Q42. Solve the equation: \(\ln(x+5) = 1 + \ln x\), giving your answer in terms of \(e\). \[3\]
### ANSWERS

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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<tbody>
<tr>
<td>Q1.</td>
<td>$-2 &lt; x &lt; \frac{1}{4}$</td>
</tr>
<tr>
<td>Q2.</td>
<td>$x = 0.438$ ($u=1.61$)</td>
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<tr>
<td>Q3. (i)</td>
<td>$a=6$ ; $b=-3$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$(x+1)(4x+1)(2x-1)$</td>
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<tr>
<td>Q4.</td>
<td>$x = 2/\sqrt{3}$</td>
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<tr>
<td>Q5. (i)</td>
<td>$a=3$</td>
</tr>
<tr>
<td>(ii)</td>
<td>(a) $(2x+1)(2x^2 - x + 2)$</td>
</tr>
<tr>
<td>(b)</td>
<td>$x &gt; -1/2$</td>
</tr>
<tr>
<td>Q6. (i)</td>
<td>$x = -2$ and $x = 2/5$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$x = -0.569$</td>
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<tr>
<td>Q7.</td>
<td>$x = 0.975$</td>
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<tr>
<td>Q8.</td>
<td>$-5 &lt; x &lt; 3/5$</td>
</tr>
<tr>
<td>Q9. (i)</td>
<td>$-\ln 4 / \ln 3$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$x = \ln 4 / \ln 6$</td>
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<tr>
<td>Q10.</td>
<td>$x = 0.860$ ($3^x = 18/7$)</td>
</tr>
<tr>
<td>Q11.</td>
<td>$y = \frac{2-e^x}{e^x-1}$ $ (e^z = \frac{y+2}{y+1})$</td>
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<tr>
<td>Q12. (i)</td>
<td>$a = 1$ , $b = -6$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\frac{1}{2}$ ; $-3/2$</td>
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<tr>
<td>Q20.</td>
<td>$x = -0.605$ $ [e^{-2x} = \frac{1}{1-\frac{1}{5-e^{2}}}]$</td>
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<td>Q21.</td>
<td>$x = 1/11$</td>
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<td>Q22.</td>
<td>$x = 22.28$</td>
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<tr>
<td>Q23.</td>
<td>$a = 12$ ; $b = -20$</td>
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<tr>
<td>Q24.</td>
<td>$-4/5 &lt; x &lt; 6$</td>
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<tr>
<td>Q25. (i)</td>
<td>$a = 11$ ; $b = 5$</td>
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<tr>
<td>(ii)</td>
<td>$(x-13)$</td>
</tr>
<tr>
<td>Q26.</td>
<td>$0.631$ and $-0.369$</td>
</tr>
<tr>
<td>Q27.</td>
<td>$x = \frac{16}{a-8}$</td>
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<tr>
<td>Q28. (i)</td>
<td>$a = -1$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$f(x) = (x + 2)(x^2 - 3x + 7)$</td>
</tr>
<tr>
<td>x= -2 is Only one real root</td>
<td>$[as \ b^2 - 4ac = -19 &lt; 0$ No real roots]</td>
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<tr>
<td>Q29.</td>
<td>Quotient = $(2x - 4)$ and R= 8</td>
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<td>Q30. (i)</td>
<td>$-2/3$ or $4/5$</td>
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<td>(ii)</td>
<td>$y = -0.161$</td>
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<td>Q31.</td>
<td>$x = 3/2$</td>
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<td>Q32. (i)</td>
<td>$a = 12$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$(3x+1)(2x-1)(2x-3)$</td>
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Q13. 

Q14. \( x = 0.866 \)

Q15. \( x = 0.0466 \) (\( u = 16/15 \))

Q16. \( x = 1.13 \) \[ \frac{x+4}{x^2} = 4 \]

Q17. \( x < 5/3 \)

Q18. (ii) \( (x - 2)(4x + 1)^2 \) 
   (iii) \( x = 2 \)
   [ \( x = -1/4 \) not possible as \( \ln(4x - 5) \) will not be defined]

Q19. \( \frac{1}{4} a < x < \frac{5}{2} a \)

Q33. \( x < -1 \) or \( x > -3/5 \)

Q34. \( y = (e^{x^3} - 1)^{-1} \)

Q35. (i) \( a = -10 \); \( b = -1 \) 
   (ii) \( R = (3x-2) \)

Q36. 3.81

Q37. (i) \( a = 4 \) 
   (ii) \((x - 2)^2(x + 4) \)
   (iii) \(\pm\sqrt{2} \) and \(\pm2i \)

Q38. \( x = 2.30 \) \( (x^2 - x - 3 = 0) \)

Q39. \( x = 1.39 \) \( (3x^2 - 2x - 3 = 0) \)

Q40. \( 2/5 < x < 4 \)

Q41. \( x = 1.14 \) \( (5^x = \frac{25}{4}) \)

Q42. \( x = \frac{5}{(e-1)} \)