A – Level - Mathematics **P_3** **<u>TRIGONOMETRY</u>**

Exercise 1 (With References)

Q1. The angles θ and ϕ lie between 0^0 and 180^0 and are such that,

$$\tan(\theta - \phi) = 3 \qquad \text{and} \qquad \tan\theta + \tan\phi = 1$$

Find the possible values of θ and ϕ. [6]

(<u>W-15 / 31 / 32/Q3</u>)
(2017 / SP-3 /Q3)

Q2. Express the equation $\tan(\theta + 45^0) - 2\tan(\theta - 45^0) = 4$ as a quadratic equation in tan θ . Hence solve this equation for $0 \le \theta \le 180^0$ [6]

(M-16 /32 / Q2)

Q3. By expressing the equation $\operatorname{cosec}\theta = 3\sin\theta + \cot\theta$ in terms of cos θ only , solve the equation for $0^0 < \theta < 180^0$ [5]

(S -16 /31 /Q3)

Q4. i) Prove the identity $\cos 4\theta - 4\cos 2\theta = 8\sin^4\theta - 3$

ii) Hence solve the equation:

$$\cos 4\theta = 4\cos 2\theta + 3 \quad \text{for} \quad 0 \le \theta \le 360^0$$ [4]

(S-16 /32 /Q5)

Q5. i) Express $\sqrt{5}\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0^0 < \alpha < 90^0$, giving the value of α correct to two decimal places.

ii) Hence solve the equation:

$$(\sqrt{5})\cos\tfrac{1}{2}x + 2\sin\tfrac{1}{2}x = 1.2 \qquad \text{for } 0 < x < 360^0$$ [3]

(S-16 / 33 / Q3)

Q6. Prove the identity : $\tan 2\theta - \tan\theta \equiv \tan\theta \sec 2\theta$

(W-16 /31/32/5(i))

Q7. Express the equation : $\sec\theta = 3\cos\theta + \tan\theta$ as a quadratic equation in $\sin\theta$. Hence solve the equation for $-90^0 \le \theta \le 90^0$ [5]

(W-16 /31/32/Q3)

Q8. Express the equation $\cot 2\theta = 1 + \tan\theta$ as a quadratic equation in $\tan\theta$. Hence solve the equation for $0^0 < \theta < 180^0$ [6]

(W-16 /33/Q3)

Q9. The angles A and B are such that :

$$\sin(A + 45^0) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B,$$

without using a calculator , find the exact value of $\tan(A - B)$ [8]

(W-15 /33/Q6)

Q10. Solve the equation :

$$7\cos x - 6\sin 2x = 0 \quad \text{for} \quad 0 \le x \le \pi$$ [5]

(S-15 /31/Q4)

Q11. i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$,

$0^0 < \theta < 90^0$, stating the exact value of R and giving the value of α correct to 2 decimal places.

ii) Hence solve the equation :

$$3\sin\theta + 2\cos\theta = 1 , \quad \text{for} \quad 0^0 < \theta < 180^0$$ [3]

(S-15/32/Q4)

Q12. Solve the equation :

$$\cot 2x + \cot x = 3 \quad \text{for} \quad 0^0 < \theta < 180^0$$ [2]

(S-15 /33 /Q3)

Q13. i) Simplify: $\sin 2\alpha . \sec\alpha$

ii) Given that $3\cos 2\beta + 7\cos\beta = 0$, find the exact value of $\cos\beta$ [3]

(S-14 /31/Q1)

Q14. i) By sketching the graph of $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation $\operatorname{cosec} x = x(\pi - x)$ has exactly two real roots in the interval $0 < x < \pi$. [3]

ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form

$$x = \frac{1+x^2 \sin x}{\pi \sin x}$$ [2]

iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β, where $\alpha < \beta$

a) Use the interative formula :

$$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$

to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

b) Deduce the value of β correct to 2 decimal places. [1]

(S-14 /31/Q8)

Q15. Solve the equation $\cos(x + 30^0) = 2\cos x$ for $-180^0 < x < 180^0$ [5]

(S-14/32/Q3)

Q16. i) Show that the equation $\tan(x - 60^0) + \cot x = \sqrt{3}$ can be written in the form $2\tan^2 x + \sqrt{3}\tan x - 1 = 0$ [3]

ii) Hence solve the equation $\tan(x - 60^0) + \cot x = \sqrt{3}$ for

$0^0 < x < 180^0$ [3]

(S-14 /33/Q3)

Q17. i) By first expanding $\sin(2\theta + \theta)$, show that :

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$ [4]

ii) Show that , after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$ the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$

iii) Hence solve the equation : $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ giving your answer to correct 3 significant figures. [4]

(W-14 /31/32/Q8)

Q18. i) Show that $\cos(\theta - 60^0) + \cos(\theta + 60^0) \equiv \cos\theta$ [3]

ii) Given that $\frac{\cos(2x-60^0)+\cos(2x+60^0)}{\cos(x-60^0)+\cos(x+60^0)} = 3$ find the exact value of cos x

[4]

(W-14 /33/Q4)

Q19. Prove that $\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta$ [3]

(W-13 /31/32/Q5(i))

Q20. i) Given that $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec} 2\theta$, show that :

$2\sin\theta + 4\cos\theta = 3$ [3]

ii) Express $2\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^0 < \alpha < 90^0$, giving the value of α correct to 2 decimal places. [3]

iii) Hence solve the equation $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec} 2\theta$ for $0^0 < \theta < 360^0$

[4]

(W-13 /33/Q7)

Q21. i) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $o < \alpha < \frac{\pi}{2}$. Giving the value of α correct to 4 decimal places. [3]

ii) Hence solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$ [4]

(S-13 /31/Q9)

Q22. i) By expanding $\cos(x + 45^0)$, express $\cos(x + 45^0) - \sqrt{2}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0^0 < \alpha < 90^0$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places.
[5]

ii) Hence solve the equation : $\cos(x + 45^0) - \sqrt{2}\sin x = 2$ for $0^0 < x < 360^0$

[4]

(S-13 /32/Q7)

Q23. i) Solve the equation : $\tan 2x = 5\cot x$, for $0^0 < x < 180^0$ [5]

ii) Express $\sqrt{3}\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact value of R and α [3]

(S-13 /33/Q3,4(i))

Q24. Solve the equation : $\operatorname{cosec} 2\theta = \sec\theta + \cot\theta$ for $0^0 < x < 360^0$ [6]

(S-12 /32/Q4)

Q25. It is given that equation $\tan 3x = k\tan x$, when k is a constant and

$\tan x \neq 0$

i) By first expanding $\tan(2x + x)$, show that :

$(3k - 1)\tan^2 x = k - 3$ [4]

ii) Hence solve the equation $\tan 3x = k\tan x$, when $k = 4$ for $0^0 < x < 180^0$

[3]

iii) Show that the equation $\tan 3x = k\tan x$ has no roots in the interval $0^0 < x < 180^0$ when $k = 2$. [1]

(S-12 /33/Q6)

Q26. Solve the equation : $\sin(\theta + 45^0) = 2\cos(\theta - 30^0)$ for $0^0 < x < 180^0$ [5]

(W-12 /31/32/Q3)

Q27. i) Express $24\sin\theta - 7\cos\theta$ in the form :

$R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^0$. Give the value of α correct to 2 decimal places. [3]

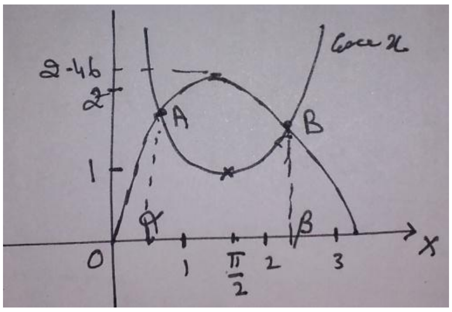
ii) Hence find the smallest positive value of θ satisfying the equation :

$24\sin\theta - 7\cos\theta = 17$ [2]

(W-12 /33/Q2)

A – Level - Mathematics **P_3** **<u>TRIGONOMETRY</u>**

Exercise 1(With References) (ANSWERS)

Q1. $\theta = 135^0$ and $\phi = 63.4^0$ Or $\theta = 53.1^0$ and $\phi = 161.6^0$	Q2. $\theta = 28.7^0$ or 165.4^0
Q3. . $\theta = 131.8^0$	Q4. . $\theta = 68.5^0$, 111.5^0 , 248.5^0 and 291.5^0
Q5. $x = 216.5^0$	Q6. Prove
Q7. $\theta = -41.8^0$	Q8. $\theta = 135^0$ or 18.4^0
Q9. $\frac{3}{11}$	Q10. 0.623 , 1.57 and 2.52 rad.
Q11. i) $\sqrt{(13)}\sin(\theta + 33.69^0)$ ii) $\theta = 130.2^0$	Q12. $x = 24.9^0$ or 98.8^0
Q13. i) $x = 2\sin\alpha$ ii) $\cos\beta = \frac{1}{3}$	Q14. i) $y = \mathrm{cosec}\, x$ and $y = x(\pi - x)$ 
Q15. $x = -66.2^0$ or 113.8^0	Q16. ii) $x = 21.6^0$ or 128.4^0
Q17. iii) $x = 0.322$, 0.799 and -1.12	Q18. ii) $\frac{1}{4}(3 - \sqrt{17})$
Q19. Proof	Q20. ii) $R = \sqrt{(20)}$, $\alpha = 63.44$ iii) 74.4^0 or 338.7^0

Q21. i) $R = 5$ and $\alpha = 0.6435$ ii) 1.80 or 5.77	Q22. i) $R = 2.236$, $\alpha = 71.57^0$ ii) 315^0 or 261.9^0
Q23. i) 40.2^0 or 139.8^0 ii) $R = 2$ and $\alpha = \frac{1}{6}\pi$	Q24. 201.5^0 or 338.5^0
Q25. ii) 16.8^0 or 163.2^0	Q26. 105.9^0
Q27. i) $R = 25$, $\alpha = 16.26^0$ ii) 59.1^0	