Q1. The angles $\theta$ and $\phi$ lie between $0^0$ and $180^0$ and are such that, 

\[ \tan (\theta - \phi) = 3 \quad \text{and} \quad \tan \theta + \tan \phi = 1 \]

Find the possible values of $\theta$ and $\phi$. 

Q2. Express the equation \( \tan (\theta + 45^0) - 2 \tan (\theta - 45^0) = 4 \) as a quadratic equation in $\tan \theta$. Hence solve this equation for $0 \leq \theta \leq 180^0$. 

Q3. By expressing the equation \( \csc \theta = 3 \sin \theta + \cot \theta \) in terms of $\cos \theta$ only, solve the equation for $0^0 < \theta < 180^0$. 

Q4. i) Prove the identity $\cos 4\theta - 4 \cos 2\theta = 8 \sin^4 \theta - 3$

ii) Hence solve the equation:

\[ \cos 4\theta = 4 \cos 2\theta + 3 \quad \text{for} \quad 0 \leq \theta \leq 360^0 \]

Q5. i) Express $\sqrt{5} \cos x + 2 \sin x$ in the form $R \cos (x - \alpha)$, where $R > 0$ and $0^0 < \alpha < 90^0$, giving the value of $\alpha$ correct to two decimal places.

ii) Hence solve the equation:

\[ (\sqrt{5}) \cos \frac{x}{2} + 2 \sin \frac{x}{2} = 1.2 \quad \text{for} \quad 0 < x < 360^0 \]

Q6. Prove the identity: $\tan 2\theta - \tan \theta \equiv \tan \theta \sec 2\theta$
Q7. Express the equation: \( \sec \theta = 3 \cos \theta + \tan \theta \) as a quadratic equation in \( \sin \theta \). Hence solve the equation for \(-90^0 \leq \theta \leq 90^0\) [5]

(W-16 /31/32/Q3)

Q8. Express the equation \( \cot^2 \theta = 1 + \tan \theta \) as a quadratic equation in \( \tan \theta \). Hence solve the equation for \(0^0 < \theta < 180^0\) [6]

(W-16 /33/Q3)

Q9. The angles A and B are such that:

\[
\sin (A + 45^0) = (2\sqrt{2}) \cos A \quad \text{and} \quad 4 \sec^2 B + 5 = 12 \tan B,
\]
without using a calculator, find the exact value of \( \tan (A - B) \) [8]

(W-15 /33/Q6)

Q10. Solve the equation:

\[ 7 \cos x - 6 \sin 2x = 0 \quad \text{for} \quad 0 \leq x \leq \pi \] [5]

(S-15 /31/Q4)

Q11. i) Express \( 3 \sin \theta + 2 \cos \theta \) in the form \( R \sin (\theta + \alpha) \), where \( R > 0 \), \(0^0 < \theta < 90^0\), stating the exact value of \( R \) and giving the value of \( \alpha \) correct to 2 decimal places.

ii) Hence solve the equation:

\[ 3 \sin \theta + 2 \cos \theta = 1 \], \quad \text{for} \quad 0^0 < \theta < 180^0 \] [3]

(S-15/32/Q4)

Q12. Solve the equation:

\[ \cot 2x + \cot x = 3 \quad \text{for} \quad 0^0 < \theta < 180^0 \] [2]

(S-15 /33/Q3)

Q13. i) Simplify: \( \sin 2\alpha \cdot \sec \alpha \)

ii) Given that \( 3 \cos 2\beta + 7 \cos \beta = 0 \), find the exact value of \( \cos \beta \) [3]

(S-14 /31/Q1)
Q14. i) By sketching the graph of \( y = \csc x \) and \( y = x (\pi - x) \) for \( 0 < x < \pi \), show that the equation \( \csc x = x (\pi - x) \) has exactly two real roots in the interval \( 0 < x < \pi \). \[3\]

ii) Show that the equation \( \csc x = x (\pi - x) \) can be written in the form \[ x = \frac{1 + x^2 \sin x}{\pi \sin x} \] \[2\]

iii) The two real roots of the equation \( \csc x = x (\pi - x) \) in the interval \( 0 < x < \pi \) are denoted by \( \alpha \) and \( \beta \), where \( \alpha < \beta \)

a) Use the iterative formula:
\[
x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}
\]
to find \( \alpha \) correct to 2 decimal places. Give the result of each iteration to 4 decimal places. \[3\]

b) Deduce the value of \( \beta \) correct to 2 decimal places. \[1\]

(S-14/31/Q8)

Q15. Solve the equation \( \cos (x + 30^0) = 2 \cos x \) for \(-180^0 < x < 180^0\) \[5\]

(S-14/32/Q3)

Q16. i) Show that the equation \( \tan (x - 60^0) + \cot x = \sqrt{3} \) can be written in the form \( 2 \tan^2 x + \sqrt{3} \tan x - 1 = 0 \) \[3\]

ii) Hence solve the equation \( \tan (x - 60^0) + \cot x = \sqrt{3} \) for \( 0^0 < x < 180^0 \) \[3\]

(S-14/33/Q3)

Q17. i) By first expanding \( \sin (2\theta + \theta) \), show that:
\[
\sin 3\theta = 3\sin \theta - 4\sin^3 \theta
\] \[4\]

ii) Show that, after making the substitution \( x = \frac{2\sin \theta}{\sqrt{3}} \), the equation \( x^3 - x + \frac{1}{6} \sqrt{3} = 0 \) can be written in the form \( \sin 3\theta = \frac{3}{4} \).
iii) Hence solve the equation: \( x^3 - x + \frac{1}{6} \sqrt{3} = 0 \) giving your answer to correct 3 significant figures. \([4]\)

(W-14/31/32/Q8)

Q18. i) Show that \( \cos(\theta - 60^0) + \cos(\theta + 60^0) \equiv \cos\theta \) \([3]\)

ii) Given that \( \frac{\cos(2x-60^0) + \cos(2x+60^0)}{\cos(x-60^0) + \cos(x+60^0)} = 3 \) find the exact value of \( \cos x \) \([4]\)

(W-14/33/Q4)

Q19. Prove that \( \cot\theta + \tan\theta = 2\cosec 2\theta \) \([3]\)

(W-13/31/32/Q5(i))

Q20. i) Given that \( \sec\theta + 2\cosec\theta = 3\cosec 2\theta \), show that:

\[ 2\sin\theta + 4\cos\theta = 3 \] \([3]\)

ii) Express \( 2\sin\theta + 4\cos\theta \) in the form \( R\cos(\theta - \alpha) \), where \( R > 0 \) and \( 0^0 < \alpha < 90^0 \), giving the value of \( \alpha \) correct to 2 decimal places. \([3]\)

iii) Hence solve the equation \( \sec\theta + 2\cosec\theta = 3\cosec 2\theta \) for \( 0^0 < \theta < 360^0 \) \([4]\)

(W-13/33/Q7)

Q21. i) Express \( 4\cos\theta + 3\sin\theta \) in the form \( R\cos(\theta - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{\pi}{2} \). Giving the value of \( \alpha \) correct to 4 decimal places. \([3]\)

ii) Hence solve the equation \( 4\cos\theta + 3\sin\theta = 2 \) for \( 0 < \theta < 2\pi \) \([4]\)
Q22. i) By expanding \( \cos(x + 45^0) \), express \( \cos(x + 45^0) - \sqrt{2} \sin x \) in the form \( R \cos(x + \alpha) \), where \( R > 0 \) and \( 0^0 < \alpha < 90^0 \). Give the value of \( R \) correct to 4 significant figures and the value of \( \alpha \) correct to 2 decimal places. [5]

ii) Hence solve the equation: \( \cos(x + 45^0) - \sqrt{2} \sin x = 2 \) for \( 0^0 < x < 360^0 \) [4]

Q23. i) Solve the equation: \( \tan 2x = 5 \cot x \), for \( 0^0 < x < 180^0 \) [5]

ii) Express \( \sqrt{3} \cos x + \sin x \) in the form \( R \cos(x - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{\pi}{2} \), giving the exact value of \( R \) and \( \alpha \) [3]

Q24. Solve the equation: \( \csc 2\theta = \sec \theta + \cot \theta \) for \( 0^0 < x < 360^0 \) [6]

Q25. It is given that equation \( \tan 3x = k \tan x \), when \( k \) is a constant and \( \tan x \neq 0 \)

i) By first expanding \( \tan(2x + x) \), show that:

\[
(3k - 1) \tan^2 x = k - 3
\]

[4]

ii) Hence solve the equation \( \tan 3x = k \tan x \), when \( k = 4 \) for \( 0^0 < x < 180^0 \) [3]

iii) Show that the equation \( \tan 3x = k \tan x \) has no roots in the interval \( 0^0 < x < 180^0 \) when \( k = 2 \). [1]
Q26. Solve the equation: \( \sin (\theta + 45^0) = 2 \cos (\theta - 30^0) \) for \(0^0 < x < 180^0\) \[5\]  

(W-12/31/32/Q3)

Q27. i) Express \(24 \sin \theta - 7 \cos \theta\) in the form:

\(R \sin (\theta - \alpha)\), where \(R > 0\) and \(0 < \alpha < 90^0\). Give the value of \(\alpha\) correct to 2 decimal places. \[3\]

ii) Hence find the smallest positive value of \(\theta\) satisfying the equation:

\(24 \sin \theta - 7 \cos \theta = 17\) \[2\]  

(W-12/33/Q2)
### A – Level - Mathematics  
#### P3  
#### TRIGONOMETRY  

**Exercise 1 (With References) (ANSWERS)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
</table>
| Q1. | θ = 135° and φ = 63.4°  
Or θ = 53.1° and φ = 161.6° |
| Q2. | θ = 28.7° or 165.4° |
| Q3. | θ = 131.8° |
| Q4. | θ = 68.5°, 111.5°, 248.5° and 291.5° |
| Q5. | x = 216.5° |
| Q6. | Prove |
| Q7. | θ = -41.8° |
| Q8. | θ = 135° or 18.4° |
| Q9. | $\tan(\theta) = \sqrt{13}$  
ii) θ = 130.2° |
| Q10. | 0.623, 1.57 and 2.52 rad. |
| Q11. | i) $\sqrt{13} \sin(\theta + 33.69°)$  
ii) θ = 130.2° |
| Q12. | x = 24.9° or 98.8° |
| Q13. | i) x = 2 sin α  
ii) $\cos\beta = \frac{1}{3}$ |
| Q14. | i) $y = \cot x$ and $y = x(\pi - x)$ |
| Q15. | x = -66.2° or 113.8° |
| Q16. | ii) x = 21.6° or 128.4° |
| Q17. | iii) x = 0.322, 0.799 and −1.12 |
| Q18. | ii) $\frac{1}{4}(3 - \sqrt{17})$ |
| Q19. | Proof |
| Q20. | ii) $R = \sqrt{20}$, α = 63.44  
iii) 74.4° or 338.7° |
<table>
<thead>
<tr>
<th>Question</th>
<th>i)</th>
<th>ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q21.</td>
<td>R = 5 and α = 0.6435</td>
<td>1.80 or 5.77</td>
</tr>
<tr>
<td>Q22.</td>
<td>R = 2.236, α = 71.57°</td>
<td>315° or 261.9°</td>
</tr>
<tr>
<td>Q23.</td>
<td>40.2° or 139.8°</td>
<td>R = 2 and α = ( \frac{1}{6} \pi )</td>
</tr>
<tr>
<td>Q24.</td>
<td>201.5° or 338.5°</td>
<td></td>
</tr>
<tr>
<td>Q25.</td>
<td>16.8° or 163.2°</td>
<td></td>
</tr>
<tr>
<td>Q26.</td>
<td>105.9°</td>
<td></td>
</tr>
<tr>
<td>Q27.</td>
<td>R = 25, α = 16.26°</td>
<td>59.1°</td>
</tr>
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