Mathematics – A Level

P$_3$

Vectors in 3D **Exercise 1**

1. Points *A*, *B*, *C* are such that $\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$; $\vec{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$; $\vec{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$

The plane *m* is perpendicular to AB and contains point C.

(i) Find Vector equation of *AB* line. **[2]**

(ii) Obtain equation of plane *m*, and give the answer in the form. $ax + by + cz = d$ **[2]**

(iii) The line *AB* interest the plane *m* at a point *N*. Find the position vector of *N* and show that *CN*= $\sqrt{13}$. **[5]**

2017/ SP-3/Q7

2. Two planes have equations: $3x + y - z = 2$ and $x - y + 2z = 3$

(i) Show that the planes are perpendicular. **[3]**

(ii) Find the vector equation of the line of intersection of the two planes. **[6]**

W-16/ 31/Q8

W-16/ 32/Q8

3. The line *l* has vector equation:

$$\boldsymbol{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$$

(i) Find the position vector of two points on the line whose distance from the origin is $\sqrt{10}$. **[5]**

(ii) The plane *p* has equation:

ax + y + z = 5, where a is constant. The acute angle between the line *l* and the plane p is equal to $\sin^{-1}(2/3)$. Find the possible value of a. **[5]**

W-16/ 33/Q10

4. The line *l* has equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and the plane p has equation $\boldsymbol{r} \bullet \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$.

(i) Show that *l* || p. **[3]**

(ii) A line m lies in plane p and is perpendicular to l. The line m passes through the point with coordinate (5,3,1). Find the vector equation of m. **M-16/ 32/Q8** **[6]**

5. The points A, B, C, D have position vectors:

$\vec{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$; $\vec{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$; $\vec{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$; $\vec{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

(i) Find the equation of plane containing points A, B and C, giving your answer in the form :

$ax + by + cz = d$ **[6]**

(ii) The line through D parallel to OA meets the plane with equation $x + 2y - z = 7$ at the point P.

Find the position vector of P and show that the length of DP is $2\sqrt{14}$. **S-16/ 31/Q9** **[5]**

6. The point A, B, C have position vectors $\vec{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$; $\vec{OB} = 4\mathbf{j} + \mathbf{k}$ and $\vec{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral ABCD is a parallelogram.

(i) Find the position vector of D and verify that the parallelogram is a rhombus. **[5]**

(ii) The plane p is parallel to OA and the line BC lies in p. Find the equation of p, giving your answer in the form $ax + by + cz = d$. **S-16/ 32/Q9** **[5]**

7. The points A and B have position vector, $\vec{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\vec{OB} = 2\mathbf{i} + 3\mathbf{k}$. The line l has vector equation: $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

(i) Show that the line passing through A and B does not intersect l. **[4]**

(ii) Show that the length of perpendicular from A to l is $1/\sqrt{2}$. **S-16/ 33/Q8** **[5]**

8. The line l_1 passes through the points (0,1,5) and (2,-2,1). The line l_2 has equation:

$\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$

(i) Show that lines l_1 and l_2 are skew. **[6]**

(ii) Find the acute angle between line l_2 and X- axis. **S-15/ 31/Q6** **[3]**

9. The points A and B have position vectors $\vec{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\vec{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$

(i) Show that l does not intersect the line passing through A and B. **[5]**

(ii) Find the equation of plane containing the line *l* and point A. Give your answer in the form. ax + by + cz = d

S-15/ 32/Q10

10. Two planes have equation :

x + 3y - 2z = 4 and 2x + y + 3y = 5. The planes intersect in the line *l*.

(i) Calculate the acute angle between the two planes. **[4]**

(ii) Find the vectors equation of the line *l*. **S-15/ 33/Q9** **[6]**

11. Two points A and B have coordinates (-1, 2, 5) and (2, -2, 11) respectively. The plane p passes through B and is perpendicular to AB :

(i) Find the equation of plane p, giving your answer in the form $ax + by + cz = d$ **[3]**

(ii) Find the acute angle between 'p' and the Y-axis. **S-11/ 31/Q3** **[4]**

12. A plane has equation 4x – y + 5y = 39. A line is parallel to the vector (**i** - 3**j** + 4**k**) and passes through point A (0, 2, -8). The line meets the plane at the point B

(i) Find the coordinates of B. **[3]**

(ii) Find the acute angle between the line and plane. **[4]**

(iii) A point C lies on the line and is such that the distance between C and B is twice the distance between A and B. Find the possible coordinates of point C. **[3]**

W-15/ 33/Q8

13. The line *l* has equation:

$$\boldsymbol{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

(i) Show that the length of the perpendicular from A to *l* is 15. **[5]**

(ii) The line lies in the plane with the equation $ax + by - 3z + 1 = 0$. Find the values of *a* and *b*. **[5]**

W-14/ 31/Q10

W-14/ 32/Q10

14. The equation of two lines are :

$$\boldsymbol{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k})$$

$$\boldsymbol{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k})$$, where a is constant.

(i) Show that lines intersect for all values of a. **[4]**

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the values of a. **[4]**

W-14/ 33/Q7

15. The line l has equation

$$\boldsymbol{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}),$$

The plane p passes through the point (4, -1, 2) and is perpendicular to l.

(i) Find the equation p in the form $ax + by + cz = a$. **[2]**

(ii) Find the perpendicular distance from origin to p. **[3]**

(iii) A second plane q is parallel to p and the perpendicular distance between p and q is 14 units. Find the possible equations of q. **S-14/ 31/Q7** **[3]**

16. The position vectors of points A, B ard C are: $\vec{OA} = i + 2j + 3k$; $\vec{OB} = 2i + 4j + k$ and

$\vec{OC} = 3i + 5j - 3k$

(i) Find the exact value of cos (BAC). **[4]**

(ii) Hence find the exact value of area of triangle ABC. **[3]**

(iii) Find the equation of plane which is parallel to the y-axis and contains the line through B and C. Give your answer in the form ax + by + cz = d. **S-14/ 32/Q10** **[5]**

17. The line l has equation $\boldsymbol{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation: $2x + 3y - 5z = 18$

(i) Find the position vector of the point of intersection of l and p. **[3]**

(ii) Find the acute angle between l and p. **[4]**

(iii) A second plane q is perpendicular to the plane p and contains the line l. find the equation of q, give your answer in the form $ax + by + cz = d$. **S-14/ 33/Q10** **[5]**

18. The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$

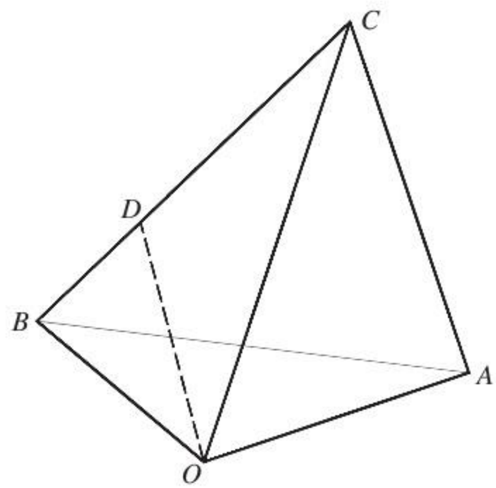
(i) Find the position vector of the point of intersection of the line through A and B and plane p. **[4]**

(ii) A second plane q has an equation of the form $x + by + cz = d$. The plane q contains the line AB, and the acute angle between the planes p and q is 60°. Find the equation of q. **[7]**

S-13/ 32/Q10

19. Given points $\vec{OA}=\begin{pmatrix}2\\-1\\2\end{pmatrix}$; $\vec{OB}=\begin{pmatrix}0\\3\\1\end{pmatrix}$; $\vec{OC}=\begin{pmatrix}3\\0\\4\end{pmatrix}$

The point D lies on BC, and is such that CD = 2DB.



(i) Find the equation of plane ABC. Give your answer in the form $ax+by+cz=d$. **[6]**

(ii) Find the position vector of D. **[1]**

(iii) Show that the length of perpendicular from A to OD is $\frac{1}{3}\sqrt{65}$. **W-13/ 32/Q9** **[4]**

20. Two planes have equations $3x-y+2z=9$ and $x+y-4z=-1$

(i) Find the acute angle between the planes. **[3]**

(ii) Find a vector equation of the line of intersection of the planes. **[6]** **W-13/ 33/Q6**

21. Given Points P and Q, $\vec{OP}=7i+7j-5k$ and $\vec{OQ}=-5i+j+k$. The midpoint of PQ is point A. The plane π is perpendicular to the line PQ and passes through A.

(i) Find the equation of π, giving answer, in the form $ax+by+cz=d$. **[4]**

(ii) The straight line through P parallel to the X- axis, meets π at the point B. Find the distance AB. **S-13/ 31/Q6** **[5]**

22. The line *l* has equation : ***r* = (i + j + k) +** λ **(a*i*** + 2***j*** + ***k***), where a is a constant. The plane p has equation $x+2y+2z=6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to plane p. **[2]**

(ii) The line l intersects the line passing through the points with position vectors 3**i + **2**j** + **k** and **i + j – k.** **[4]**

(iii) The acute angle between the line l and plane p is $\tan^{-1}2$. **S-13/ 33/Q10** **[5]**

23. The point P has coordinates (-1, 4, 11) and the line l has equation. $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

(i) Find the perpendicular distance from P to l. **[4]**

(ii) Find the equation of the plane which contains P and l, give your answer in the form $ax + by + cz = d$. **S-12/ 31/Q8** **[5]**

24. Two planes m and n have equations : $x + 2y - 2z = 1$ and $2x - 2y + z = 7$ respectively. The line l has equation ***r* = (i + j - k) +** λ **(**2**i** + **j** + 2**k**)

(i) Show that l is parallel to m. **[3]**

(ii) Find the position vector of the point of intersection of l and n. **S-12/ 32/Q10** **[3]**

(iii) A point P lies on l is such that its perpendicular distances from m and n are equal. Find the position vectors of two possible positions for P and calculate the distance between them. **[6]**

25. The lines l and m have equations. ***r* = (**3**i -** 2**j** + **k) +** λ **(-i** + 2**j** + **k**) and ***r* = (**4**i +** 4**j** + 2**k) +** μ **(**a**i** + b**j - k**), where a and b are constants.

(i) Given l and m intersect, show that $2a - b = 4$. **[4]**

(ii) Given l and m are perpendicular . Find the values of a and b. **S-12/ 33/Q9** **[4]**

(iii) when a and b have these values find the point of intersection of l and m. **[2]**

26. The A, B, C are such that: $\vec{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$; $\vec{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$; $\vec{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}$

The plane m is parallel to $\vec{OC}$ and contains A and B.

(i) Find the equation of m, give your answer in the form $ax + by + cz = d$ **[6]**

(ii) Find the length of the perpendicular from C to the line through A and B. **[5]**

W-12/ 31/Q10

W-12/ 32/Q10

27. Two lines have equations: $\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$. and $\mathbf{r} = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$, where p is a constant.

It is given that lines intersect.

(i) Find the values of p and the coordinates of the points of intersection. **[5]**

(ii) Find the equation of the plane containing the two lines, giving your answer in the form $ax + by + cz = d$ where a, b, c and d are the integers. **[5]**

W-12/ 33/Q8

28. Given lines l_1 and l_2:

l_1: $\mathbf{r} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) + s\,(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and l_2: $\mathbf{r} = (4\mathbf{i} + 6\mathbf{j} + \mathbf{k}) + t\,(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

(i) Prove that l_1 and l_2 are coplanar. **[4]**

(ii) find the equation of plane containing l_1 and l_2. **S-10/ 31/Q10** **[5]**

29. The plane p has equation 2x -3y + 6z = 16. The plane q is parallel to p and contains the point with position vector **(i** + 4**j** + 2**k)**

(i) Find the equation of q, giving your answer in the form $ax + by + cz = d$ **[2]**

(ii) Calculate the perpendicular distance between p and q. **[3]**

(iii) The line *l* is parallel to plane p and also parallel to the plane with equation x – 2y + 2z = 5. Given *l* passes through origin find the vector equation of *l*. **[5]**

(iv) Find the distance between the planes:

2x – 3y + 6z = 16

W-09/ 32/Q10

4x – 6y +12z + 4 = 0 **[3]**

30. Find the equation of the plane passing through the intersection of the planes
2x + 3y – z + 1 = 0 and x + y – 2z + 3 = 0 and

(i) perpendicular to the plane 3x – y – 2 z – 4 = 0. **[5]**

(ii) passing through a point (1, -2, 3). **[4]**

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Mathematics – A Level

P_3

Vectors in 3D **Exercise 1 Answer Sheet**

1.	(i) $\boldsymbol{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ (ii) $2x - 2y + z = 4$ (iii) $\vec{ON} = \frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ and $CN = \sqrt{13}$	8.	(i) l_1: $\boldsymbol{r} = (2\lambda,\ \ 1-3\lambda,\ \ 5-4\lambda)$ l_2: $\boldsymbol{r} = (7+\mu,\ \ 1+2\mu,\ \ 1+5\mu)$ Show (a) line l_1 and l_2 are not parallel. (b) Equate the respective component- solve two equations for λ and $\mu \Rightarrow \lambda = 2$ and $\mu = -3$ but these values does not satisfy the third equation. (ii) $\theta = 79.5^{\circ}$ or 1.39 radian
2.	(i) Show that a scalar product of normal vector is zero. (ii) $\boldsymbol{r} = 7\mathbf{i} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$ Or $= (\mathbf{i} + \mathbf{k}) + \lambda(-\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$	9	(i) line AB; $\boldsymbol{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ Line l ; $\boldsymbol{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ Equate the respective component of AB and l. Solve first two equations to get $\lambda = 0; \mu = 1$ but these values does not satisfy the third equation (ii) $x + 4y + 7z = 19$
3.	(i) $-\mathbf{i} + 3\mathbf{j}$ and $\frac{7}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$ (ii) a = ± 2	10.	(i) Angle between the two planes is the angle between their normal. $\theta = 85.9^{\circ}$ or 1.50 radian (ii) $\boldsymbol{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$
4.	(i) Show $l \perp$ Normal or $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 0$ (ii) $\boldsymbol{r} = 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$	11.	(i) $3x - 4y + 6z = 80$ (ii) 30.8°
5.	(i) $5x - 2y + 3z = 5$ (ii) $\vec{OP} = (-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$	12.	(i) (3, -7, 4) (ii) 54.8° or 0.956 radian (iii) (-3, 11, -20) ; (9, -25, 28)
6.	$\vec{OA} = 3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and verify adjacent sides AB = BC. (ii) $-7x + 8y - 3z = 29$	13.	(i) 15 (ii) a = 2, b = -2
7.	(i) equation of line AB: $\boldsymbol{r} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ Equate the respective components of line AB and line l. Solve first two equations, $\lambda = -1, \mu = 2$ but these values does not satisfy the third equation. (ii) $\frac{1}{\sqrt{2}}$	14.	(i) Equate the components of points on both the lines: Solve first two; satisfy the third $\lambda = a, \mu = 1$ (ii) a =3 and -2

15.	(i) $2x - 3y + 6z = 23$ (ii) distance = 23/7 (iii) $2x - 3y + 6z = 121$ or $2x - 3y + 6z = -75$	25.	(i) 2a – b = 4 (ii) a = 3 ; b = 2 (iii) (**i** + 2**j** + 3**k**)
16.	(i) Cos $\angle BAC = \frac{20}{21}$ (ii) Area of triangle = $\frac{1}{2}\sqrt{41}$ (iii) 4x + z = 9	26.	(i) 3x + z = 13 (ii) $3\sqrt{2}$ (4.24)
17.	(i) $(-\frac{1}{2}$ **i** + 3**j** - 2**k**) (ii) 23.2° (or 0.404 radian) (iii) 4x + 19y + 13z = 29	27.	(i) p = 9 ; (7, -1, 2) (ii) 11x – 10y – 7z = 73
18.	(i) $\frac{13}{2}$ **i** + - $\frac{3}{2}$ **j** + $\frac{1}{2}$ **k** (ii) x- 4y – z = 12	28.	(i) Intersect for s = -1 and t = -2 (ii) -5x + 3y + 4z = 2
19.	(i) 3x + y – 2z = 1 (ii) $\vec{OD} =$ **i** + 2**j** + 2**k**	29.	(i) 2x – 3y + 6z = 2 (ii) 2 (iii) ***r*** **=** λ **(-6i - 2j + k)** (iv) 18/7
20.	(i) 68.7° (or 1.18 radian) (ii) ***r*** **= (2i - 3j) +** λ **(i + 7j + 2k)** or ***r*** **= (-17j - 4k) +** λ **(i + 7j + 2k)**	30.	(i) 7x + 13y + 4z = 9 x + 3y +4z = 7
21.	(i) 2x + y – z = 8 (ii) B (-2, 7, -5) and AB = 5.20		
22.	(i) a = -6 (ii) a = 4 (iii) a = 0 and a = 60/31		
23.	(i) $\sqrt{104}$ or 10.2 (ii) 3x – 9y + z = -28		
24.	(i) Show $l \perp$ normal to m. (ii) 5**i** + 3**j** + 3**k** (iii) $\vec{OP} =$ (7**i** + 4**j** + 5**k**) and $\vec{OP'} =$ (3**i** + 2**j** + **k**) Required distance $PP' = 6$		