Normal Distribution
Revision

S.1

Probability and Statistics 1

S.20 M.20 S.20 M.19 S.19 W.19

Suresh Goel
(Fomer Director)
Alliance World School,
Noida, Delhi - NCR,
INDIA.
Example 1: A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(a) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres.

The daily sales at another petrol station are \( X \) litres, where \( X \) is normally distributed with mean \( m \) and standard deviation 560. It is given that \( P(X > 8000) = 0.122 \)

(b) Find the value of \( m \).

(c) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days.

\[
\text{Solution: } \mu = 4520, \sigma = 560, x = 3900
\]

(a) \[ P(X > x) = P(z > \frac{x - \mu}{\sigma}) = P(z > \frac{3900 - 4520}{560}) = P(z > -1.07) \]

\[ = P(z < 1.07) = \Phi(1.07) = 0.8567 \]

(b) Now \( \mu = m, \sigma = 560, P(X > 8000) = 0.122 \Rightarrow P(x < 8000) = 1 - 0.122 \)

Now \( P(x < 8000) = P(\frac{8000 - m}{560}) = 0.878 \)

\[ \Rightarrow m = 8350 \times \frac{1.165}{560} \]

\[ \Rightarrow m = 1435 \]

(c) Now \( P(x > 8000) = 0.122 \) and \( p = 1 - 0.122 = 0.878 \); \( n = 6 \)

\[ P(\sum x < 2) = P(0) + P(1) \]

Using Binomial Distribution:

\[ P(x) = \binom{n}{x} p^x (1-p)^{n-x} \]

\[ = 0.878^6 + 6 \times 0.878^5 \times 0.122 \]

\[ = 0.4581 + 0.3819 \]

\[ = 0.840 \]
Example 2: A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6 is thrown 90 times.

(a) Use an approximation to find the probability that a 3 is obtained fewer than 18 times.

(b) Justify your use of the approximation in part (a).

On another occasion, the same die is thrown repeatedly until a 3 is obtained.

(c) Find the probability that obtaining a 3 requires fewer than 75 tries.

Solution: \( p = \frac{1}{6}, \quad q = \frac{5}{6}; \quad \text{mean} \mu = np = 90 \times \frac{1}{6} = 15 \)

\[ X \sim B(n, p) \Rightarrow N(\mu, \sigma^2) \]

\[ \sigma^2 = npq = 90 \times \frac{1}{6} \times \frac{5}{6} = 25 \Rightarrow \sigma = \sqrt{25} = 5 \]

\[ P(X < 18) = P(Z < \frac{17.5 - 15}{\sqrt{15}}) \]

\[ = P(Z < 0.7071) \]

\[ = 0.760 \]

\[ \text{Continuity Correction} \]

(b) \( np = 15 > 5 \) and \( nq = 75 > 5 \), so the normal distribution is justified.

(c) \( P(\text{get a 3}) = \frac{1}{6}, \quad q = \frac{5}{6} \)

\[ P(X < 7) = P(X \leq 6) = 1 - q^6 \]

\[ = 1 - \left(\frac{5}{6}\right)^6 \]

\[ = 0.665 \]
Example 3: The weights of apples of a certain variety are normally distributed with mean 82 grams. 32% of the apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples. 

(b) Find the probability that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams.

Solution:

(a) $P(X > 87) = P\left( \frac{X - 82}{\sigma} > \frac{87 - 82}{\sigma} \right) = 0.22 \quad \left[ \text{Find } \frac{X - \mu}{\sigma} = \frac{82}{\sigma} \right]

\Rightarrow P\left( Z < \frac{5}{\sigma} \right) = 1 - 0.22 = 0.78

\Rightarrow \frac{5}{\sigma} = \phi^{-1}(0.78) = 0.742

\Rightarrow \sigma = \frac{5}{0.742} \approx 6.756

(b) $|X - \mu| < 4 \Rightarrow -4 < X - \mu < 4

\Rightarrow -\frac{4}{\sigma} < \frac{X - \mu}{\sigma} < \frac{4}{\sigma} \quad \left( \frac{Z = \frac{X - \mu}{\sigma}}{\sigma} \right)

P\left( \frac{|X - \mu|}{\sigma} < 4 \right) = P\left( -\frac{4}{\sigma} < Z < \frac{4}{\sigma} \right)

= P\left( -0.6176 < Z < 0.6176 \right) \quad \left[ \frac{y = \frac{4}{\sigma} = 0.6176}{\sigma} \right]

= \phi(0.6176) - (1 - \phi(0.6176))

= 2\phi(0.6176) - 1

= 2 \times 0.74137 - 1

= 0.463$
Example 5: In Greenon, 70% of the adults own a car. A random sample of 120 adults from Greenon is chosen, use an approximation to find the probability that more than 75 of them own a car.

Solution: Mean, \( \mu = np = 120 \times 0.7 = 84 \), Var = npq = 120 \times 0.7 \times 0.3 = 25.2

\[ P(X > 75) \approx P(X > 75.5) \approx P(X > 75.5) \]

Using continuity correction:

\[ P(Z > \frac{75.5 - 84}{\sqrt{25.2}}) \approx P(Z > -0.693) \]

\[ Z = \frac{X - \mu}{\sigma} \]

\[ = 0.955 \]

Example 5: The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

(a) Find the prob. that a randomly chosen female snake of this species has length between 50 cm and 60 cm.

The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 300 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find the estimate of the mean and standard deviation of the lengths of male snakes of this species.

Solution: \( \bar{x} = 54, \sigma = 6.1 \)

\[ P(50 < X < 60) = P\left(\frac{50 - 54}{6.1} < Z < \frac{60 - 54}{6.1}\right) \]

\[ = P(-0.6557 < Z < 0.9836) \]

\[ = \phi(0.9836) - \phi(-0.6557) \]

\[ = \phi(0.9836) - [1 - \phi(0.9836)] \]

\[ = 0.8375 + 0.7441 - 1 \]

\[ = 0.582 \]

Note: \( x = 0.16 < 0.5 \)

\[ \phi^{-1}(0.16) = -\phi^{-1}(1 - 0.16) \]

Conclusion: \( \mu = 49.6, \sigma = 4.65 \)
Example 6: Trees in the Redland forest are classified as tall, medium or short according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 d.p.

(c) Find the height above which trees are classified as tall.

\[
\begin{align*}
\text{Solution:} & \quad \mu = 40, \sigma = 12, \\
& \quad (a) \quad P(X < 25) = P\left(Z < \frac{25-40}{12}\right) = P(Z < -1.25) = 1 - \Phi(1.25) \\
& \quad P(\text{Short tree}) = 1 - 0.8944 = 0.106 \checkmark \\
& \quad (b) \quad P(\text{tall or medium}) = 1 - P(\text{short}) = 1 - 0.1056 = 0.8944 \checkmark \\
& \quad P(\text{tall}) = \frac{1}{3} P(\text{tall or medium}) = \frac{1}{3} \times 0.8944 = 0.298 \checkmark \\
& \quad (c) \quad P(\text{tall}) = 0.298 \\
& \quad \text{let height of trees above which considered tall is } h \\
& \quad P(X > h) = P\left(Z > \frac{h-40}{12}\right) = 0.298 \\
& \quad \Rightarrow P\left(Z < \frac{h-40}{12}\right) = 1 - 0.298 = 0.702 \\
& \quad \Rightarrow \frac{h-40}{12} = \Phi^{-1}(0.702) = 0.53 \\
& \quad \frac{h-40}{12} = 0.53 \\
& \quad \Rightarrow h = 46.4 \text{ m} \checkmark
\end{align*}
\]
Example 7: On a given day, the probability that Moena messages her friend Pasta is 0.72. Use an approximation to find the probability that in any period of 100 days, Moena messages Pasta on fewer than 64 days. \( n = 100 \)

Solution: \( p = 0.72, \ q = 0.28 \)

\[ \mu = np = 100 \times 0.72 = 72 \]

\[ \sigma^2 = npq = 100 \times 0.72 \times 0.28 \]

\[ \text{B}(n, p) \approx N(\mu, \sigma^2) \]

\[ \phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

\[ P(X < 64) = \int_{-\infty}^{64} \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx \]

**Continuity correction:**

\[ X < 63.5 \]

\[ 1 - \phi(1.893) = 1 - 0.9708 \approx 0.0292 \]

Example 8: In a certain town, the time, X hours, for which people watch television in a week, has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 81 hours in a week.

\( n = 81 \)

Solution: mean \( \mu = 15.8 \), \( \sigma = 4.2 \)

\[ P(X < 21) = \phi \left( \frac{21 - 15.8}{4.2} \right) \]

\[ = \phi (1.238) \]

\[ = 0.892 \]

(b) Find the value of \( k \) such that \( P(X < k) = 0.75 \)

\[ P(X < k) = \phi \left( \frac{k - 15.8}{4.2} \right) = 0.75 \]

\[ k - 15.8 = \phi^{-1}(0.75) = 0.6744 \]

\[ k = 18.6 \]

(18.6)
Example 9: A pair of fair coins is thrown 80 times.
Use an approximation to find the prob. that pair of tails is obtained more than 25 times.

\[ S = \{HH, HT, TH, TT\}, \quad P(HT) = \frac{1}{4} = 0.25 \]
\[ p = 0.25, \quad q = 0.75; \quad \mu = np = 80 \times 0.25 = 20 \]
\[ \sigma^2 = npq = 80 \times 0.25 \times 0.75 = 15 \]
\[ P(X > 25) = P \left( Z > \frac{25.5 - 20}{\sqrt{15}} \right) = P(Z > 1.42) \]
\[ \text{a continuity correct is rep.} \]
\[ x > 25 \sim x \geq 26 \]
\[ = 1 - P(Z < 1.42) \]
\[ = 1 - \phi(1.42) \]
\[ = 1 - 0.9222 = 0.0778 \]

Example 10: The times taken, in minutes, for trains to travel between Alchester and Becton are normally distributed with mean 140 and standard deviation 12.

(i) Find the prob. that a randomly chosen train will take less than 132 minutes to travel between Alchester and Becton.

(ii) The prob. that a randomly chosen train takes more than \( k \) minutes to travel between Alchester and Becton is 0.675.

Find \( k \).

Solution: \( \mu = 140, \sigma = 12 \)

(i) \( P(X < 132) = \Phi(\frac{132 - 140}{12}) \)
\[ = \Phi(-0.667) \]
\[ = 1 - \Phi(0.667) \]
\[ = 1 - 0.7477 \]
\[ = 0.252 \]

(ii) \( P(X > k) = P \left( Z > \frac{k - 140}{12} \right) = 0.675 \)
\[ \frac{k - 140}{12} = \Phi^{-1}(0.325) \]
\[ = -\phi^{-1}(1 - 0.675) \]
\[ = -\phi^{-1}(0.325) \]
\[ = -1.041 \]
\[ \frac{k - 140}{12} = -1.041 \]
\[ k = 134.6 \]

\( \Phi^{-1}(a) \) if \( a < 0.5 \) \( \Rightarrow \Phi^{-1}(a) = -\phi^{-1}(1 - a) \)
Example 11: The results of a survey by a large supermarket show that 35% of its customers shop online. For a random sample of 100 customers, use a suitable approximating distribution to find the probability that more than 39 shop online.

Solution: \[ n = 100, \ p = 0.35, \ \text{Var} = 0.35 \times 0.65 \times 100 = 22.75 \]

\[ P(X > 39) = P\left( \frac{Z > 39.5 - 35}{\sqrt{22.75}} \right) \]

\[ = P(Z > 0.943) \]

\[ = 1 - P(Z < 0.943) \]

\[ = 1 - 0.8286 \]

\[ = 0.1714 \]

Example 12: In a country the prob. that child owns a bicycle is 0.65. A random sample of 250 children from the country is chosen. Use a suitable approximation to find the prob. that fewer than 179 own a bicycle.

Solution: \[ n = 250, \ p = 0.65, \ \text{Var} = npq = 250 \times 0.65 \times 0.35 \]

\[ P(X < 179) = P\left( Z < \frac{178.5 - 162.5}{\sqrt{56.875}} \right) \]

\[ = P(Z < 2.12) \]

\[ = 0.983 \]
Example 13: The weights of adult female giraffes has a normal distribution with mean 830 kg and standard deviation 120 kg. 

(i) There are 430 adult female giraffes in a particular reserve. Find the number of these adult female giraffes which can be expected to weigh less than 700 kg. -- [4]

(ii) Given that 90% of adult female giraffes weigh between $(830 - W)$ and $(830 + W)$ kg. Find the value of $W$. -- [13]

The weight of adult male giraffes has a normal distribution with mean 1190 kg and standard deviation 5 kg.

(iii) Given that 83.4% of adult male giraffes weigh more than 950 kg. Find the value of $W$. -- [80/61/8/7] -- [8]

\[
\begin{align*}
\text{Solution: mean } &= 830, \sigma = 120 \\
(i) & \quad P(X < 700) = P(Z < \frac{700 - 830}{120}) \\
& = P(Z < -1.083) \\
& = 1 - 0.8606 \\
& = 0.1394 \\
& \Rightarrow n = \frac{430 \times 0.1394}{0.95} = 60 \\
(ii) & \quad P(830 - W < X < 830 + W) \\
& = P(\frac{830 - W - 830}{120} < Z < \frac{830 + W - 830}{120}) \\
& = P(-W < Z < \frac{W}{120}) = 0.999 \\
& \Rightarrow P(Z < \frac{W}{120}) - P(Z < -\frac{W}{120}) = 0.99 \\
& \Rightarrow P(Z < \frac{W}{120}) - (1 - P(Z < \frac{W}{120})) = 0.99 \\
& \Rightarrow 2P(Z < \frac{W}{120}) = 1.99 \\
& \Rightarrow 2P(Z < \frac{W}{120}) = 1.09 \\
& \Rightarrow \frac{W}{120} = 1.09 \\
& \Rightarrow W = 120 \times 1.09 \\
& \Rightarrow W = 197.4 \\
& \Rightarrow \sigma = 247 \\
\end{align*}
\]
Example 14: The volume of ink in a certain ink cartridge has a normal distribution with mean 30 ml and standard deviation 1.5 ml. People in an office use a total of 8 cartridges of this ink per month. Find the expected number of cartridges per month that contain less than 28.9 ml of this ink.

Solution: \( \mu = 30 \), 3.d \( \sigma = 1.5 \)

\[
P(X < 28.9) = P\left( Z < \frac{28.9 - 30}{1.5} \right) = P\left( Z < -0.733 \right) = 1 - \Phi(0.733)
\]

\[
= 1 - 0.7682 = 0.2318.
\]

Number of expected cartridges: \( np = 8 \times 0.2318 = 1.85 \)

Expected Number: 2

Example 15: It is known that 20% of male giant pandas in a certain area weigh more than 121 kg, and 71.9% weigh more than 102 kg. Weights of male giant pandas in this area have a normal distribution. Find the mean and standard deviation of the weight of male giant pandas in this area.

Solution: \( P(X > 121) = P\left( Z > \frac{121 - \mu}{\sigma} \right) = 0.2 \)

\[
= 1 - P\left( Z < \frac{121 - \mu}{\sigma} \right) = 1 - 0.2 = 0.8
\]

\[
\frac{121 - \mu}{\sigma} = \Phi^{-1}(0.8) = 0.842
\]

\[
\Rightarrow 121 - \mu = 0.842 \sigma \quad \text{(1)}
\]

Also, \( P(X > 102) = P\left( Z > \frac{102 - \mu}{\sigma} \right) = 0.719 \)

\[
= 1 - P\left( Z < \frac{102 - \mu}{\sigma} \right) = 1 - 0.719 = 0.281
\]

\[
\Rightarrow 102 - \mu = \Phi^{-1}(0.281) = -\Phi^{-1}(1 - 0.281) = -\Phi^{-1}(0.719) = -0.58
\]

\[
\Rightarrow 102 - \mu = -0.58 \sigma \quad \text{(2)}
\]

Solve \( 0 \) & \( 2 \)

\[
\sigma = 13.4 \quad \text{and} \quad \mu = 110
\]

\[
\Phi'(a) = -\Phi'(1-a) \quad \text{if} \ a < 0.5
\]
Example 16: The time taken in minutes, by a ferry to cross a lake, has a normal distribution with mean 85 and standard deviation 6.8.

(i) Find the prob. that, on a randomly chosen occasion, the time taken by the ferry to cross the lake is between 79 and 91 minutes.

\[
P(79 < X < 91) = P(79 - 85 < Z < 91 - 85) = P(Z > -\frac{6.8}{6.8}) - P(Z < \frac{0}{6.8}) = 0.96 - 0.4 = 0.56
\]

(ii) Over a long period it is found that 96% of ferry crossings take longer than a certain time \( t \) minutes. Find the value of \( t \).

\[
P(Z < -t) = 0.04 \Rightarrow \Phi(-t) = 0.04 \Rightarrow t = 73.1 \text{ min}
\]

Example 17: On average, 36% of the people who go to a particular theatre are men. Use an approximation to find the prob. that, in a random sample of 100 people who go to the theatre, fewer than 70 are men.

\[
p = 0.34, \quad q = 0.66, \quad \mu = np = 60 \times 0.34 = 20.4, \quad \sigma = \sqrt{npq} = \sqrt{60 \times 0.34 \times 0.66} = 4.3564
\]

\[
P(X < 70) = P(Z < \frac{70 - 20.4}{4.3564}) = P(Z < -1.2496) = 1 - \Phi(-1.2496) = 1 - 0.8944 = 0.106
\]
Example 18: The shortest time recorded by an athlete in a 400 m race is called their personal best (PB). The PBs of the athletes in a large athletics club are normally distributed with mean 49.2 seconds and standard deviation 2.8 seconds.

(i) Find the prob. that a randomly chosen athlete from this club has a PB between 46 and 53 seconds.

(ii) It is found that 92% of athletes from this club have PBs of more than t seconds. Find the value of t.

Three athletes from the club are chosen at random.

(iii) Find the prob. that exactly 2 have PBs less than 46 seconds.

Solution: mean \( \mu = 49.2 \), std dev \( \sigma = 2.8 \)

(i) \( P(46 < x < 53) = P\left(\frac{46-49.2}{2.8} < Z < \frac{53-49.2}{2.8}\right) = P\left(-1.143 < Z < 1.357\right) = \phi(1.357) - \phi(-1.143) = 0.9126 - 0.1056 = 0.808 \checkmark \)

(ii) \( P(x > t) = P(z > t - 49.2) = 0.92 \)

\[ \Rightarrow P(z < t - 49.2) = 1 - 0.92 = 0.08 \]

\[ \Rightarrow \frac{t - 49.2}{2.8} = \phi^{-1}(0.08) = -1.0419 \]

\[ \Rightarrow t - 49.2 = -1.0419 \times 2.8 \]

\[ \Rightarrow t = 48.3 \checkmark \]

\( \phi^{-1}(a) = -\phi^{-1}(1-a) \) if \( a < 0.5 \),

Using Binomial Prob.

\( P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \)
Example 19. In a Quadrant, 66% of households are satisfied with the speed of their Wi-Fi connection.

A random sample of 150 households in Quadrant is chosen, use suitable approximation to find the prob. that more than 84 are satisfied with the speed of their Wi-Fi connection. \[ \frac{\text{expected mean}}{\text{expected variance}} \]

\[ n = 150 \]
\[ \mu = np = 150 \times 0.66 = 99 \]
\[ \sigma^2 = np(1-p) = 150 \times 0.66 \times 0.34 = 33.66 \]

\[ \varphi(X > 84) = \varphi\left( z > \frac{84 - 5 - 99}{33.66} \right) \]
\[ = \varphi(z > -2.499) \]
\[ = 1 - \varphi(z < -2.499) \]
\[ = 1 - 0.0068 \]
\[ = 0.9932 \]

\[ B(n, p) \sim N(99, 33.66) \] (continued)

Example 20: The height in metres of six trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

(i) Find the prob. that a six tree chosen at random in this forest has a height less than 45 metres.

(ii) Find the prob. that a six tree chosen at random in this forest has a height within 5 metres of the mean.

In another forest, the heights of another type of six trees are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

(iii) Find the mean and standard deviation of the height of trees of this type.
20(iii): \( P(X < 10) = \frac{48}{500} = 0.096 \)
  \[ P(X < 10) = P\left( Z < \frac{10 - \mu}{\sigma} \right) = 0.096 \]
  \[ \Rightarrow \frac{10 - \mu}{\sigma} = \Phi^{-1}(0.096) = \Phi^{-1}(1 - 0.904) = -1.30 \]
  \[ \Rightarrow 10 - \mu = -1.30 \sigma \] — (1)

Now \( P(X > 24) = \frac{74}{500} = 0.152 \)
  \[ P(X > 24) = P(Z > \frac{24 - \mu}{\sigma}) = 0.152 \]
  \[ \Rightarrow P\left( Z < \frac{24 - \mu}{\sigma} \right) = 1 - 0.152 = 0.848 \]
  \[ \Rightarrow \frac{24 - \mu}{\sigma} = \Phi^{-1}(0.848) = 1.028 \]
  \[ \Rightarrow 24 - \mu = 1.028 \sigma \] — (2)

Solving (1) & (2) \( \sigma = 6 \) and \( \mu = 17.8 \) (3)

Example 21: The heights of students at the Mainland College are normally distributed with mean 148 cm and standard deviation 8 cm.

(i) The prob. that a mainland student chosen at random has a
  height less than 140 cm, is 0.617. Find the value of \( \mu \). --[3]
  120 Mainland students are chosen at random.

(ii) Find the number of these students that would be expected
    to have a height within half a standard deviation of the mean.--[4]

\( \text{Solution: Mean } \mu = 148, \text{ s.d } \sigma = 8 \)

(i) \( P(X < 8) = P\left( Z < \frac{148 - \mu}{8} \right) = 0.67 \)
  \[ \Rightarrow \frac{148 - \mu}{8} = \Phi^{-1}(0.67) = 0.44 \]
  \[ \Rightarrow 148 - \mu = 0.44 \times 8 \]
  \[ \Rightarrow \mu = 151.52 \text{ or } 152 \]

(ii) \( P(48 – 4 < X < 148 + 4) \)
  \[ \frac{48 - 148}{8} = \Phi^{-1}(0.67) = 0.44 \]

\( \Rightarrow P\left( -0.5 < Z < 0.5 \right) = 2 \Phi(0.5) - 1 \)

\( = 2 \Phi(0.5) - 1 \)

\( \Rightarrow \eta = 120 \times 0.383 \)

\( \Rightarrow \eta = 45.96 \) (or 46)
Example 22: A competition is taking place between two choices, the Notes and the Classics. There is a large audience for the competition.

- 30% of audience are Notes supporters.
- 45% of audience are Classics supporters.
- The rest of the audience are not supporters of either of these choices.
- No one in the audience supports both of these choices.

A random sample of 240 people is chosen from the audience. Use a suitable approximation to find the probability that fewer than 50 do not support either of the choices. \[ W \sim B(240, 0.75) \]

Solution:

\[
P(\text{do not support either}) = 1 - (0.30 + 0.45) = 0.25, \quad q = 0.75
\]

\[ p = 0.25, \quad n = 240, \quad \mu = np = 240 \times 0.25 = 60 \]

\[ \sigma^2 = npq = 240 \times 0.25 \times 0.75 = 45 \]

\[
P(X < 50) = P \left( Z < \frac{49.5 - 60}{\sqrt{45}} \right)
\]

\[
\phi(-1.565)
\]

\[ = 1 - \phi(1.565) = 1 - 0.9412 = 0.0588
\]

\[ X < 50 \sim X < 49.5 \]

for Normal.