Pure Math. 1

Coordinate Geometry
Circles
Exercise

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1. A diameter of a circle $C_1$ has end points at $(-3, 5)$ and $(7, 3)$.
   (a) Find an equation of the circle $C_1$. 
   The circle $C_1$ is translated by $(8, 4)$ to give circle $C_2$, as shown in the diagram. 
   (b) Find an equation of the circle $C_2$. 
   The two circles intersect at points $R$ and $S$. 
   (c) Show that the equation of the line $RS$ is $y = -2x + 13$. 
   (d) Hence show that the $x$-coordinates of $R$ and $S$ satisfy the equation $5x^2 - 60x + 159 = 0$. 
   $\Rightarrow 17 - 20/12/0/0$

2. The coordinates of the points $A$ and $B$ are $(-1, -2)$ and $(7, 4)$ respectively.
   (a) Find the equation of circle $C$, for which $AB$ is a diameter. 
   (b) Find the equation of tangent, $T$, to circle $C$ at the point $B$. 
   (c) Find the equation of the circle which is the reflection of circle $C$ in the line $T$. 
   $\Rightarrow 5-10/1/1/0/0$

3. The equation of a circle with centre $C$ is $x^2 + y^2 - 8x + 4y - 5 = 0$.
   (a) Find the radius of the circle and the coordinates of $C$. 
   The point $P(1, 2)$ lies on the circle. 
   (b) Show that the equation of the tangent to the circle at $P$ is $4y = 3x + 5$. 
   The point $Q$ also lies on the circle and $PQ$ is parallel to the $x$-axis. 
   (c) Write down the coordinates of $Q$. 
   The tangent to the circle at $P$ and $Q$ meet at $T$. 
   (d) Find the coordinates of $T$. 
   $\Rightarrow 5-10/12/1/1/0/1/0$
4. (a) The coordinates of two points A and B are (-7,3) and (5, -1) respectively. Show that the equation of the perpendicular bisector of AB is \( 3x + 2y = 11 \).

(b) A circle passes through A and B and its centre lies on the line \( 12x - 5y = 70 \). Find the equation of the circle.

5. Find the coordinates of the middle point of the chord which the circle \( x^2 + y^2 + 4x - 2y - 3 = 0 \) cuts off on the line \( x - y + 2 = 0 \).

6. Prove that the line \( x + y = 5 \) touches the circle \( x^2 + y^2 - 2x - 4y + 3 = 0 \). Find the point of contact.

7. Find the equation of a circle with radius 5, whose centre lies on the x-axis and passes through the point (2, 3).
Coordinate Geometry (Circles)

Exercise

1. (a) Centre \((-\frac{3+7}{2}, -\frac{5+3}{2})\) = \((2, -1)\)
   \[x^2 = [2 - (-3)]^2 + \left(-1 - (-5)\right)^2 = 41\]
   Equation of circle \((x-2)^2 + (y+1)^2 = 41\) \(\checkmark\)

   (b) Centre of Circle \(C_2\),
   \((2, -1) + \left(\frac{8}{4}\right) = (10, 3)\)
   some radius \(r^2 = 41\)
   Equation of circle \(C_2\)
   \((x-10)^2 + (y-3)^2 = 41\) \(\checkmark\)

(C) RS is the line of intersection of
   Circle (1) and (2)
   \((3, 1)\)
   RS and \(C_2\)  
   \(s\)
   at \(M\)
   \(M\) is the midpoint of \(RS\) and \(C_2\)
   \(M(2 + \frac{10}{2}, -\frac{1+3}{2}) = (6, 1)\)
   gradient of \(C_1C_2\) = \(\frac{3+1}{10-2} = \frac{1}{2}\)
   gradient of \(RS\) = \(-2\)
   Equation of line \(RS\)
   \[y - 1 = -2(x-6)\]
   \[y = -2x + 13\] \(\checkmark\)

(d) \(R\) and \(S\) are the intersection of \(\checkmark\)
   \((x-10)^2 + (-2x + 13 - 3)^2 = 41\)
   \[x^2 - 20x + 100 + 4x^2 - 4x + 100 = 41\]
   \[5x^2 - 20x + 159 = 0\] \(\checkmark\)

1. (a) Centre \((-\frac{1+7}{2}, -\frac{2+4}{2}) = (3, 1)\)
   \[r = \sqrt{(3+1)^2 + (1+2)^2} = 5\]
   Equation of circle \((x-3)^2 + (y-1)^2 = 25\) \(\checkmark\)

(b) Tangent \(T\) at \(B\).

   \(T \perp BC\)

   \((3, 1)\)
   \((7, 4)\)
   \[y = -\frac{4}{3}(x - 7)\]
   \[3y + 40 = 0\]

(C) \(B\) is the midpoint of \(RS\) and \(C_2\)

\(C_2\) \((11, 7)\)
   \(\text{Radius} = 5\)

New circle \(E\) \(x^2 + y^2 = 8x + 4y - 5 = 0\)
   3(a) \(x^2 + y^2 = 8x + 4y - 5 = 0\)
   \[2a = 8, 2b = 4, c = -5\]
   Centre \((-2, -2)\) \(\checkmark\)
   \[x = \sqrt{a^2 + b^2} = \sqrt{8 + 4} = 5\]

(b) \((1, 2), C(4, -2)\)
   \(\text{Grad of } PC = -\frac{3}{2}\)
   \(\text{Grad of tangent } PT = 3\)
   \(E\) \(\text{tangent } PT\)
   \[y - 2 = \frac{3}{2}(x-1) = 4y = 3x + 5\) \(\checkmark\)

(C) \(y^2 + y^2 = 8x + 4y - 5 = 0\)
   \(\text{On circle } D\) and \(y > a\)
   \[a = \sqrt{x^2 + y^2 - 8x + 4y - 5} = 0\]
   \[x^2 - 8x + 17 = 0\]
   \[x = 1, y\]
   Point on \(P(1, 2), Q(7, 2)\)
   \(\text{Grad of } CQ = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{2} = 1\)
   \(y - 2 = 2(x - 7)\)

(d) \(\text{Grad of } TQ = \frac{1}{2}\)
   \(\text{Grad of } TQ\) \(\checkmark\)
4. Mid point of \( AB \), \( M(-1, 2) \)
   \( (a) \) Gradient of \( AB = \frac{5}{2} \)
   \[ \Rightarrow \text{Grad. of } AB = \frac{-1}{3} \]
   i. Eqn. of perpendicular bisector of \( AB \)
   \[ y = -\frac{1}{3}(x+1) \]
   \[ \Rightarrow 3x + 2y = 11 \]
   \( (b) \) Centre lies on line \( 12x - 5y = 40 \)
   and centre also lies on the perpendicular bisector of chord \( AB \) of required circle

6. Given line \( x + y = 5 \) or \( y = (5-x) \)
   Circle \( x^2 + y^2 - 2x - 4y + 3 = 0 \)
   Solving \( D \) \& \( D' \)
   \[ x^2 + (5-x)^2 - 2x - 4(5-x) + 3 = 0 \]
   \[ \Rightarrow x = \frac{2}{5} \]
   from \( D \) \[ y = 5 - x = 3 \]
   i. Line \( O \) intersects circle \( G \) at exactly one point \( (2, 3) \), hence is tangent. Point \( G \), contact \( (2, 3) \)

7. Curve lies on \( x \)-axis, let \( x = 0 \)
   Centre \( C(0, 0) \)
   Passes through \( A(2, 3) \)
   \[ \text{Eqn. of circle} \]
   \[ (x-2)^2 + (y-3)^2 = 25 \]
   \[ \Rightarrow (x-2)^2 + (0-3)^2 = 25 \]
   \[ \Rightarrow a = 6 \text{ or } -2 \]
   Curve \( G(6, 6) \) or \( C(-2, 0) \)
   \[ a = 5 \]

6. Centre \( C(-2, 1) \)
   Eqn. of chord \( AB \)
   \[ x + y + 3 = 0 \]
   \[ \text{Grad. of chord } AB = 1 \]
   Grad. of perpendicular to \( CM \)
   \[ \text{Chord } = -1 \]
   From curve \( G \)
   Eqn. of chord \( CM \)
   \[ y - 1 = -1(x+2) \]
   \[ \Rightarrow x + y + 1 = 0 \]
   Solve \( D \) \& \( D' \)
   \[ M(-\frac{3}{2}, -\frac{7}{2}) \]